

Inexact relative-error proximal point algorithms

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Based on joint works with

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The problem

- Monotone inclusion:

$$\text{find } z \in \mathcal{H} \text{ such that } 0 \in T(z)$$

where $T : \mathcal{H} \rightrightarrows \mathcal{H}$ is a maximal monotone operator.

- $T : \mathcal{H} \rightrightarrows \mathcal{H}$ is a *monotone operator* if

$$\langle z - z', v - v' \rangle \geq 0 \quad \forall v \in T(z), v' \in T(z').$$

- $T : \mathcal{H} \rightrightarrows \mathcal{H}$ is a *maximal monotone operator* if T is monotone and there exists no monotone operator $S : \mathcal{H} \rightrightarrows \mathcal{H}$ such that $G(S)$ properly contains $G(T)$.

The proximal point method

- Monotone inclusion:

$$\text{find } z \in \mathcal{H} \text{ such that } 0 \in T(z)$$

where $T : \mathcal{H} \rightrightarrows \mathcal{H}$ is maximal monotone.

- Resolvent computation:

$$z_+ = (\lambda T + I)^{-1}z \iff 0 \in \lambda T(z_+) + z_+ - z.$$

- Rockafellar (1976):

$$\|z_k - (\lambda_k T + I)^{-1}z_{k-1}\| \leq e_k, \quad \sum_{k=1}^{\infty} e_k < \infty.$$

The hybrid proximal extragradient (HPE) method

- Decoupling:

$$z_+ = (\lambda T + I)^{-1}z \iff \exists v \in T(z_+), \quad \lambda v + z_+ - z = 0.$$

- Note that, in this case,

$$z_+ = z - \lambda v.$$

- HPE iteration (Solodov-Svaiter): $\sigma \in [0, 1)$,

$$v_k \in T(\tilde{z}_k), \quad \|\lambda_k v_k + \tilde{z}_k - z_{k-1}\| \leq \sigma \|\tilde{z}_k - z_{k-1}\|,$$

$$z_k = z_{k-1} - \lambda_k v_k.$$

- If $\sigma = 0$, then $z_k = \tilde{z}_k = (\lambda T + I)^{-1}z_{k-1}$.

The hybrid proximal extragradient (HPE) method

- ▶ More general version with enlargements (Solodov-Svaiter):

$$v_k \in T^{\varepsilon_k}(\tilde{z}_k), \quad \|\lambda_k v_k + \tilde{z}_k - z_{k-1}\|^2 + 2\lambda_k \varepsilon_k \leq \sigma^2 \|\tilde{z}_k - z_{k-1}\|^2,$$

$$z_k = z_{k-1} - \lambda_k v_k.$$

- ▶ For $\varepsilon \geq 0$,

$$T^\varepsilon(z) := \{v \in \mathcal{H} \mid \langle z - z', v - v' \rangle \geq -\varepsilon \quad \forall v' \in T(z')\}.$$



$$T^\varepsilon(z) \supset T(z).$$

The hybrid proximal extragradient (HPE) method

- ▶ Monteiro, Ortiz and Svaiter (2014): Numerical experiments on large scale conic semidefinite programming problems ($\sigma = 0.99$).
- ▶ Eckstein and Yao (2018): Douglas-Rachford and ADMM relative-error HPE-type algorithms. Numerical experiments on LASSO and Logistic Regression ($\sigma = 0.90$; $\sigma = 0.99$).
- ▶ Some special instances of the HPE method/framework: Forward-backward, Tseng's modified forward-backward, Korpelevich, ADMM.

The hybrid proximal extragradient (HPE) method

- ▶ Monteiro and Svaiter (2010): Iteration-complexity; global $\mathcal{O}(1/\sqrt{k})$ *pointwise* and $\mathcal{O}(1/k)$ *ergodic* convergence rates.
- ▶ Is it possible to obtain a global $\mathcal{O}(1/k)$ *pointwise* rate?
- ▶ A., Monteiro and Svaiter (2016): **Regularized** HPE method with $\mathcal{O}(\rho^{-1} \log(\rho^{-1}))$ *pointwise* iteration-complexity.

Main reference



M. Marques Alves, R. D. C. Monteiro, and B. F. Svaiter.

Regularized HPE-type methods for solving monotone inclusions with improved pointwise iteration-complexity bounds.

SIAM J. Optim., 26(4):2730–2743, 2016.

Thank you for your attention!

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