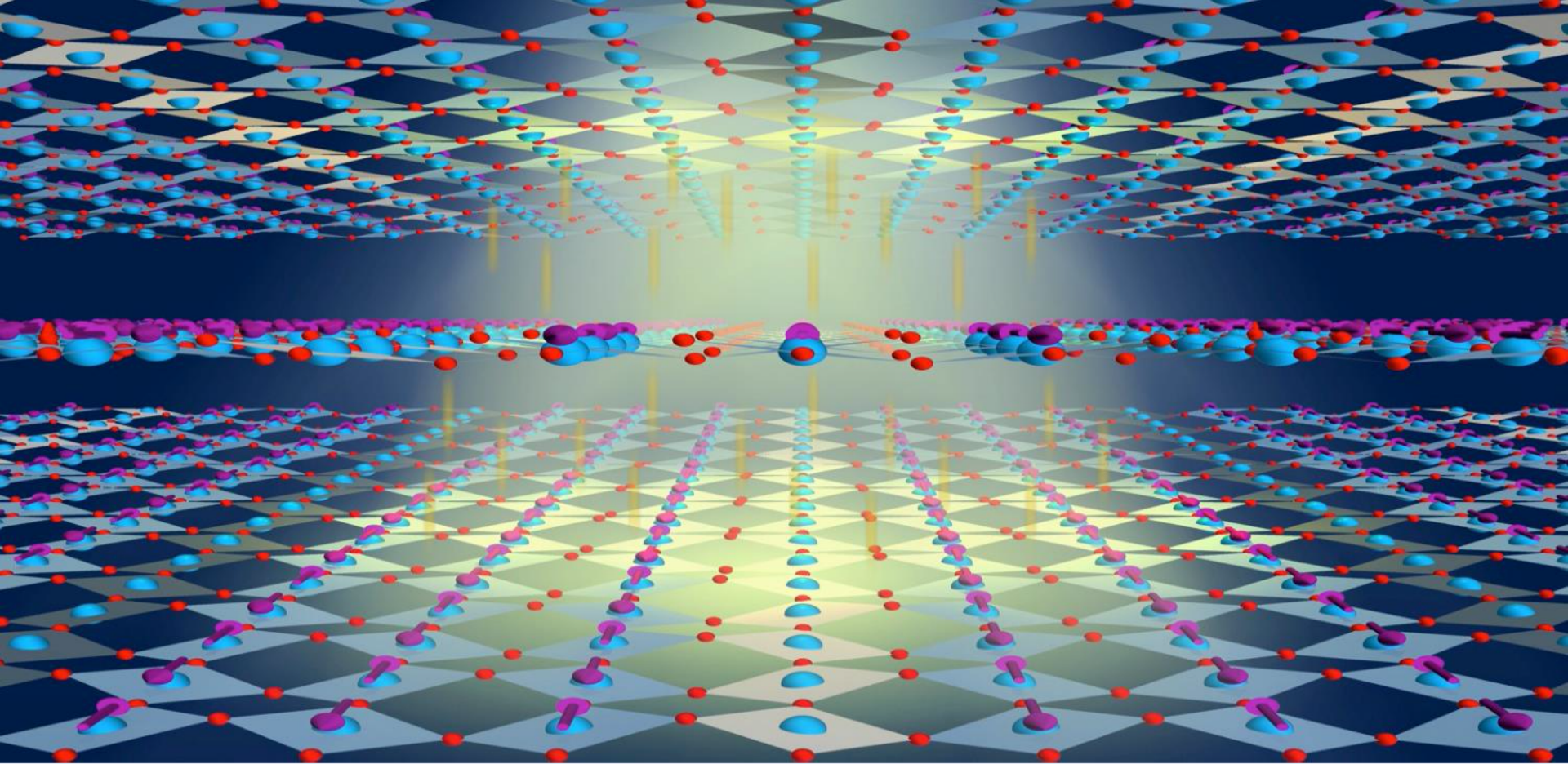
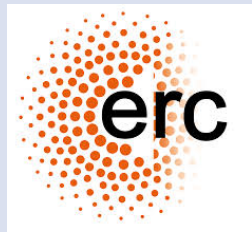




Non-equilibrium dynamics in Quantum Matter



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13 January 2020



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→ MathWorks

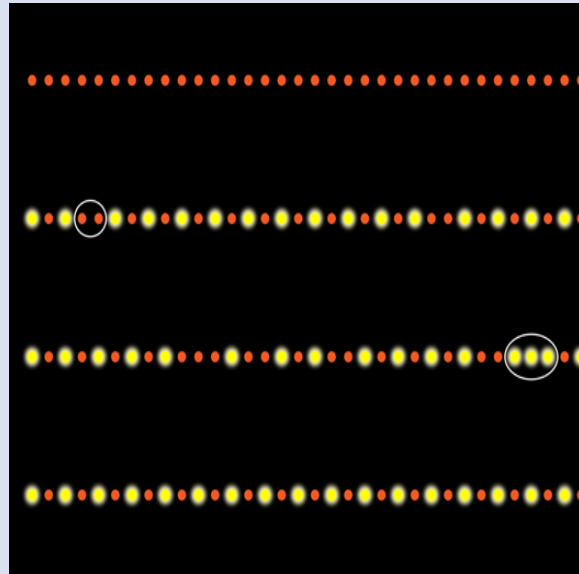
- *Heat-induced Long-Range η -Pairing in the Hubbard Model*, J. Tindall, B. Buca, J. R. Coulthard, D. Jaksch, *Physical Review Letters* **123**, 030603 (2019).
- *Complex coherent quantum many-body dynamics through dissipation*, B. Buca, J. Tindall, D. Jaksch, *Nature Communications* **10**, 1730 (2019).
- *Quantum Synchronisation Enabled by Dynamical Symmetries and Dissipation*, J. Tindall, C. Sanchez-Munoz, B. Buca, D. Jaksch, *arXiv:1907.12837*, to appear in *NJP*.

Many-Body localisation



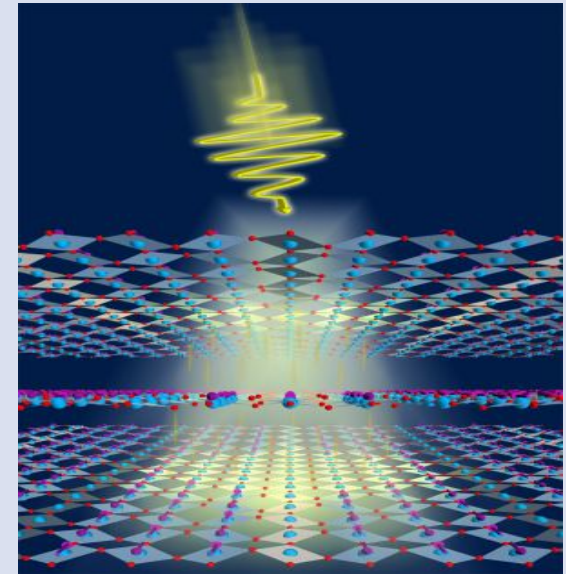
Many-Body Localization Needs a Bath, APS Physics Viewpoint, Physics **9**, 76 (2016)

Quantum scarring



Cold Atoms Bear a Quantum Scar, APS Physics Viewpoint, Physics **11**, 105 (2018)

Light-Induced Superconductivity

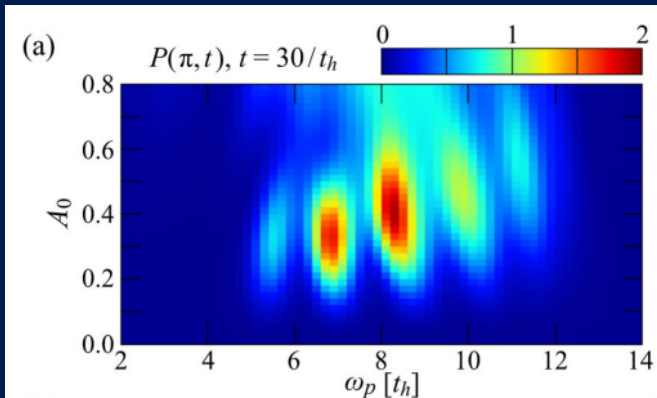


Light-Induced Superconductivity in a Stripe-Ordered Cuprate, D. Fausti et al, Science **331**, 189 (2011)

Driving

$$H \rightarrow H + f(t)$$
$$f(t + T) = f(t)$$

Pump Amplitude



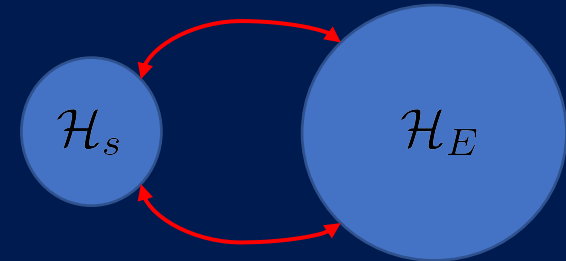
Pump Frequency

- *T. Kaneko et al, PRL* **122**, 077002 (2019)

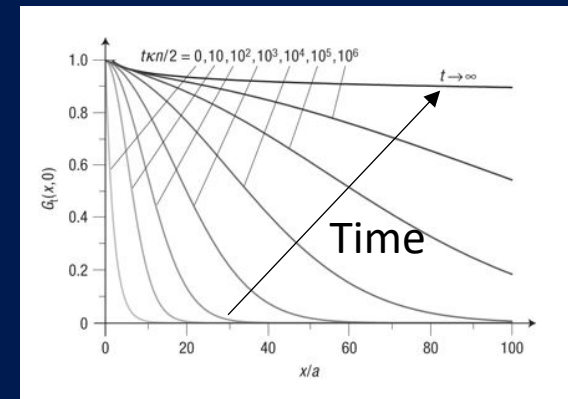
More refs...

- *J. Coulthard et al, PRB* **96**, 085104 (2017)
- *M. Sentef et al, PRB* **93**, 144506 (2016)

Dissipation



Correlations



Distance

- *S. Diehl et al, Nat. Phys* **878** (2008)

More refs...

- *B. Kraus et al, PRA* **78**, 042307 (2008)

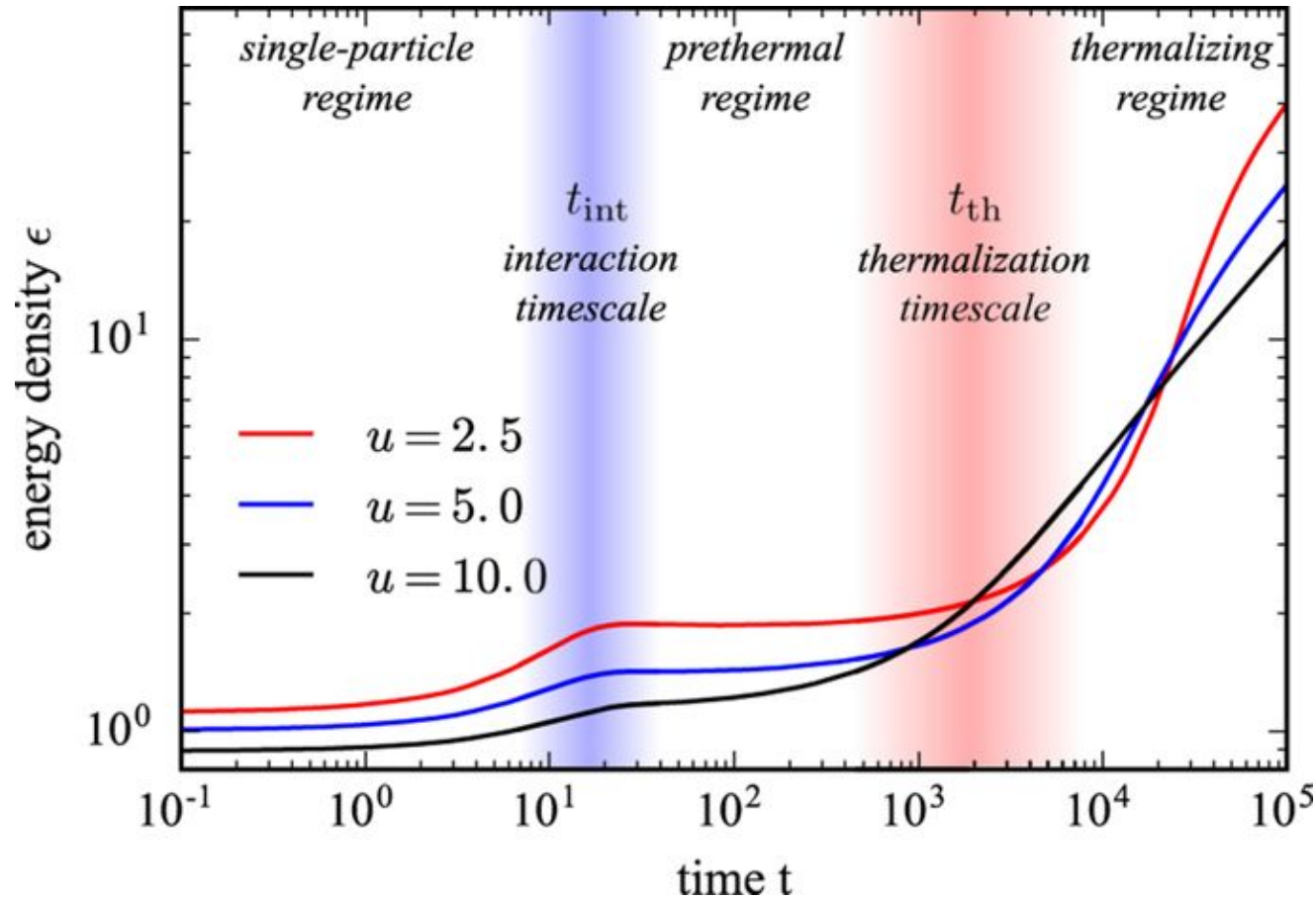
Possible light-induced superconductivity in K_3C_{60} at high temperature, M. Mitrano et al, Nature **530**, 461 (2016)

Light-Induced Superconductivity in a Stripe-Ordered Cuprate, D. Fausti et al, Science **331**, 189 (2011)

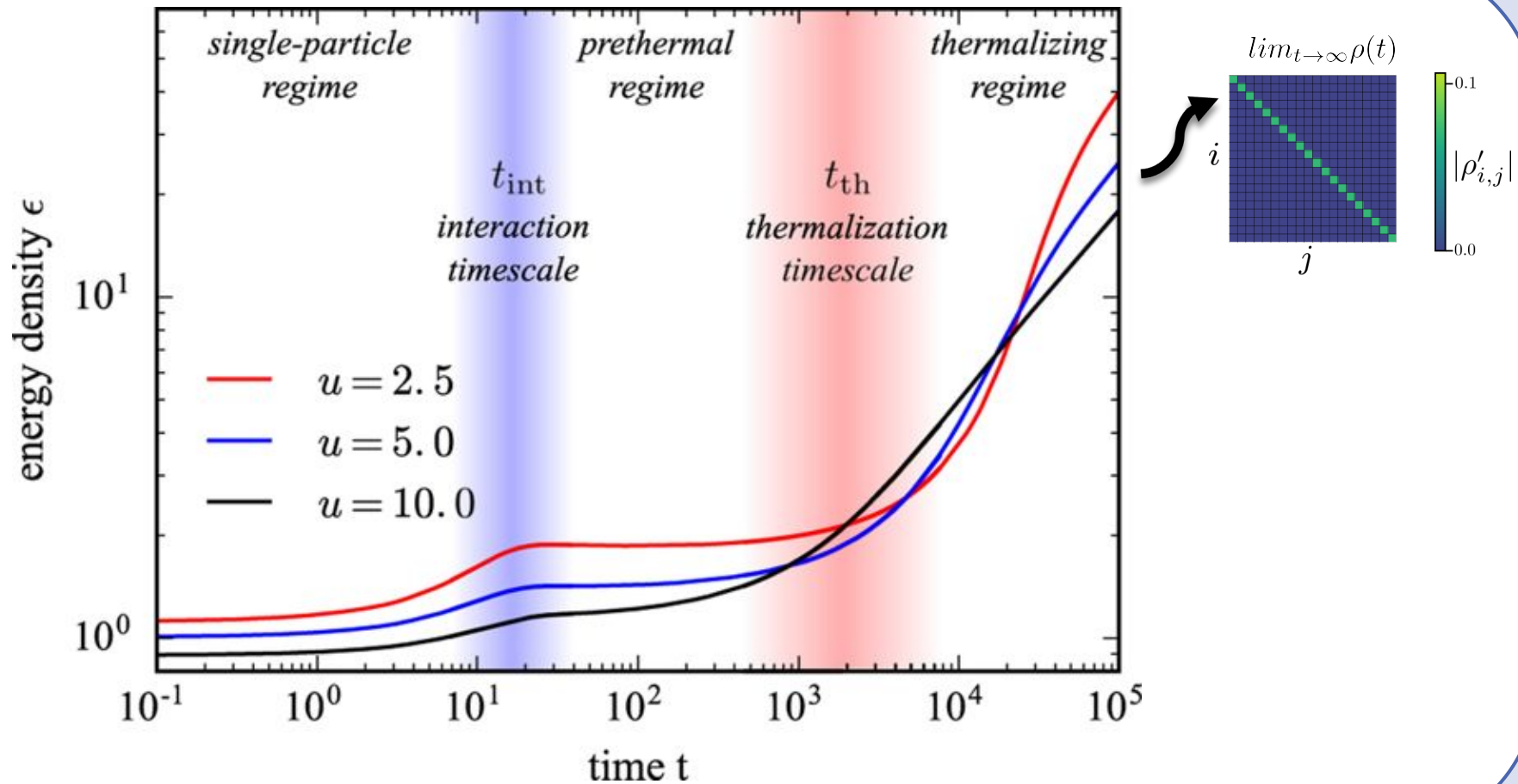
Optically enhanced coherent transport in $YBa_2Cu_3O_{6.5}$ by ultrafast redistribution of interlayer coupling, W. Hu et al, Nat. Mat. **13**, 705 (2014)

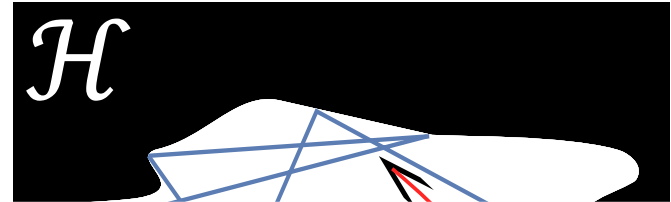
- Photo-excitation of materials can induce superconducting order
- Numerous examples in different setups: see *Photo-induced superconductivity*, A. Cavalleri, *Contemp. Phys.* **59**, 1 (2018) for a review
- Underpinned by the competition between different types of order?

Generic long-time driven dynamics



Generic long-time driven dynamics





Combination of symmetry and dissipation
→ prevent ETH
→ non-ergodic



Can we engineer a many-body quantum system that shows “perpetual” non-equilibrium dynamics?

Eigenvalues λ_i of the dynamics

$$\rho_i = e^{\lambda_i} \rho_i$$

$$\dot{\rho} = \mathcal{L}\rho = -i[H, \rho]$$

$$+ \sum_{\mu} (2 L_{\mu} \rho L_{\mu}^{\dagger} - L_{\mu}^{\dagger} L_{\mu} \rho - \rho L_{\mu}^{\dagger} L_{\mu})$$

Closed system

- purely imaginary λ_i
- dephasing

Open system

- negative real parts
- dissipation

$\text{Re}\{\lambda_i\}$



gap

ρ_{ss}

$\text{Im}\{\lambda_i\}$

Generic condition for



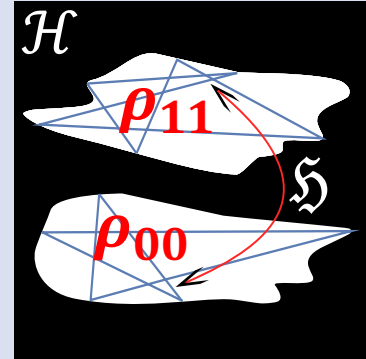
$$[H, A] = -\lambda A \quad \text{and} \quad [A, L_{\mu}^{\dagger}] = [A, L_{\mu}] = 0 \quad \forall \mu$$

- If there exists a lowering operator A with the following properties

$$[H, A] = -\lambda A \quad \text{and} \quad [A, L_\mu^\dagger] = [A, L_\mu] = 0 \quad \forall \mu$$

with $\lambda \neq 0$ then ρ_{nm} of the form (with integer $m, n > 0, m \neq n$)

$$\rho_{nm} = A^n \rho_{00} (A^\dagger)^m$$

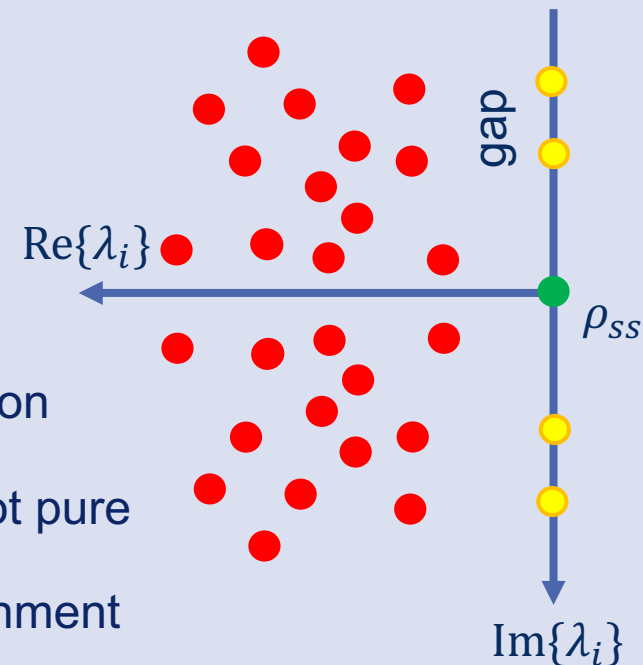


is non-stationary with $\mathcal{L}\rho_{nm} = i(m - n)\lambda\rho_{nm}$

- The mixed coherences are not necessarily decoupled from the environment

$$L_\mu \rho_{nm} L_\mu^\dagger \neq 0$$

- A dark Hamiltonian $\mathfrak{H}$ describes this long-time evolution
 - $\mathfrak{H}$ is not necessarily Hermitian, eigenstates not pure
 - evolution by $\mathfrak{H}$ not decoupled from the environment



- We consider Hermitian quantum jump operators L_μ .
- The identity operator $\mathbb{I}$ is then a stationary state of the dynamics

$$\mathcal{L}\mathbb{I} = -i[H, \mathbb{I}] + \sum_{\mu} (2 L_{\mu} \mathbb{I} L_{\mu}^{\dagger} - L_{\mu}^{\dagger} L_{\mu} \mathbb{I} - \mathbb{I} L_{\mu}^{\dagger} L_{\mu}) = 0$$

- The stationary state is a function of the conserved quantities only.
- We assume thermalization of the state within each symmetry sector

$$\rho(\tau \rightarrow \infty) \propto \exp\left(\sum_i \beta_i C_i\right)$$

where C_i are the operators describing the conserved quantities.

- Their expectation values are fixed by the Lagrange multipliers β_i .
- This is an infinite temperature state if H is not conserved.

A dissipative Hubbard model

- Hubbard Hamiltonian $H_{Hub} = H_{hop} + H_U + H_\mu + H_B$

$$H_{Hub} = -t \sum_{\langle i,j \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i + \frac{B}{2} \sum_i (n_{i\uparrow} - n_{i\downarrow})$$

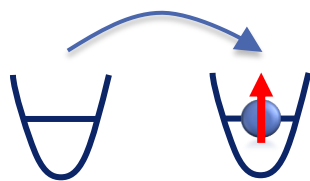
Including dissipation

$$\dot{\rho} = \mathcal{L}\rho = -i[H_{Hub}, \rho] + \sum_\mu (2 L_\mu \rho L_\mu^\dagger - L_\mu^\dagger L_\mu \rho - \rho L_\mu^\dagger L_\mu)$$

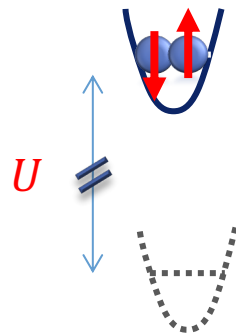
- External driving

$$H_B \rightarrow H_B(\tau) = \frac{1}{2} \sum_i B_i(\tau) (n_{i\uparrow} - n_{i\downarrow})$$

tunnelling
 t



interaction

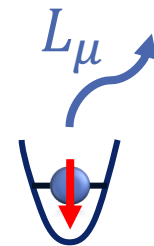


Magnetic field



$$U \gg B \approx t$$

dissipation



- Starting from a generic master equation

$$\dot{\rho} = \mathcal{L}\rho = -i[H_{\text{Hub}}, \rho] + \sum_{\mu} (2 L_{\mu} \rho L_{\mu}^{\dagger} - L_{\mu}^{\dagger} L_{\mu} \rho - \rho L_{\mu}^{\dagger} L_{\mu})$$

Dissipation in the spin sector

$$L_{\mu} \propto S_{\mu}^z$$

off-diagonal long-range
correlations between η pairs

→ **superconducting state**

Dissipation in the charge sector

$$L_{\mu} \propto n_{\mu}$$

prevent stationarity and
eigenstate thermalization

→ **non-stationarity**

- Hubbard Hamiltonian on a D-dimensional bipartite lattice in magnetic field B

$$H_{\text{Hub}} = -t \sum_{\langle i,j \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i + \frac{B}{2} \sum_i (n_{i\uparrow} - n_{i\downarrow})$$

- Spin symmetry

$$S^Z = \sum_j S_j^Z, \quad S_j^Z = \frac{1}{2} (n_{j,\uparrow} - n_{j,\downarrow}),$$

$$S^+ = \sum_j S_j^+, \quad S_j^+ = c_{j,\uparrow}^\dagger c_{j,\downarrow}$$

$$S^- = \sum_j S_j^-, \quad S_j^- = c_{j,\downarrow}^\dagger c_{j,\uparrow}$$

- These operators fulfil

$$[H_{\text{Hub}}, S^Z] = 0, \quad [H_{\text{Hub}}, S^\pm] = \pm B S^\pm$$

- Hubbard Hamiltonian on a D-dimensional bipartite lattice in magnetic field B

$$H_{\text{Hub}} = -t \sum_{\langle i,j \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i + \frac{B}{2} \sum_i n_{i\uparrow} - n_{i\downarrow}$$

- η -pairing symmetry

$$\eta^z = \frac{1}{2} \sum_j (n_j - 1),$$

$$\eta^+ = \sum_j \tau(j) \eta_j^+, \quad \eta_j^+ = c_{j,\uparrow}^\dagger c_{j,\downarrow}^\dagger,$$

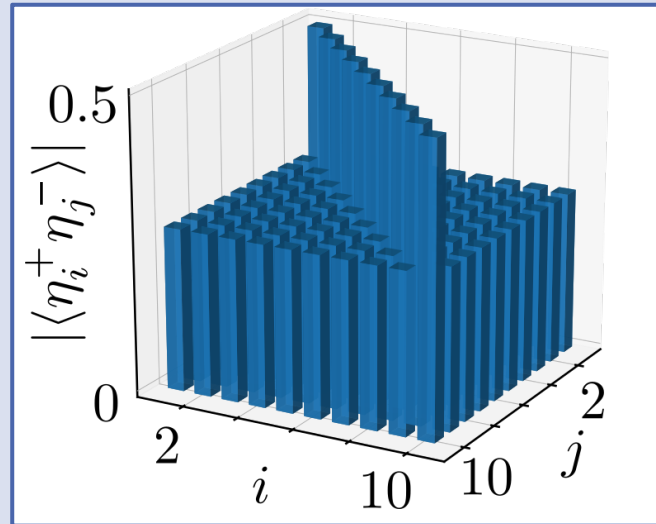
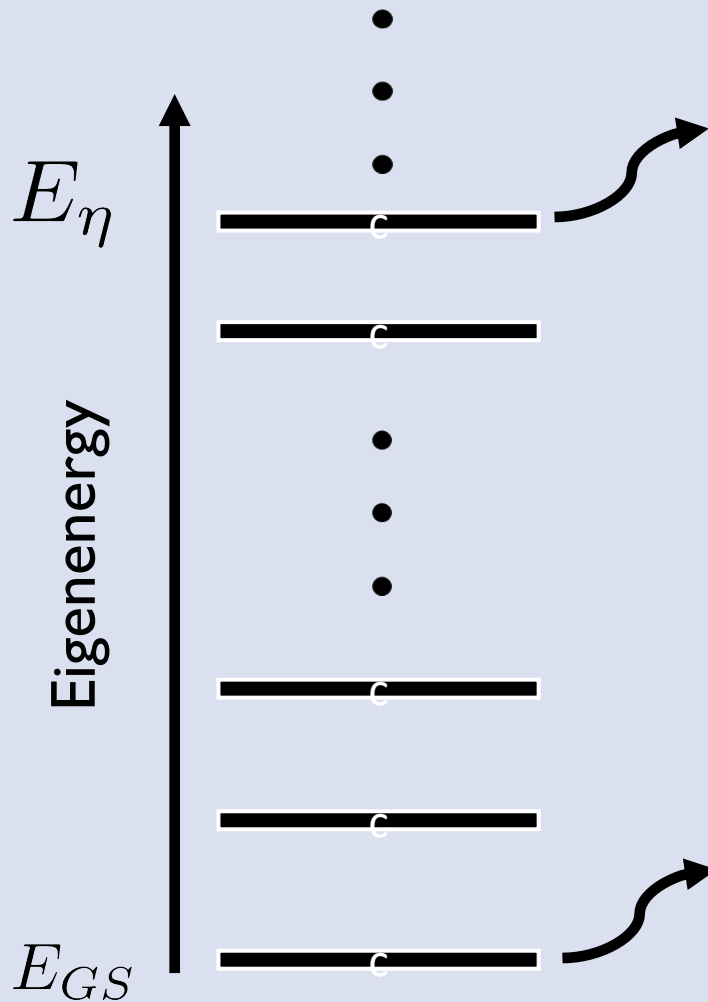
$$\eta^- = \sum_j \tau(j) \eta_j^-, \quad \eta_j^- = c_{j,\downarrow} c_{j,\uparrow},$$

with $\tau(j)$ a checkerboard pattern. They fulfil

$$[H_{\text{Hub}}, \eta^z] = 0, \quad [H_{\text{Hub}}, \eta^\pm] = \pm 2\mu \eta^\pm$$

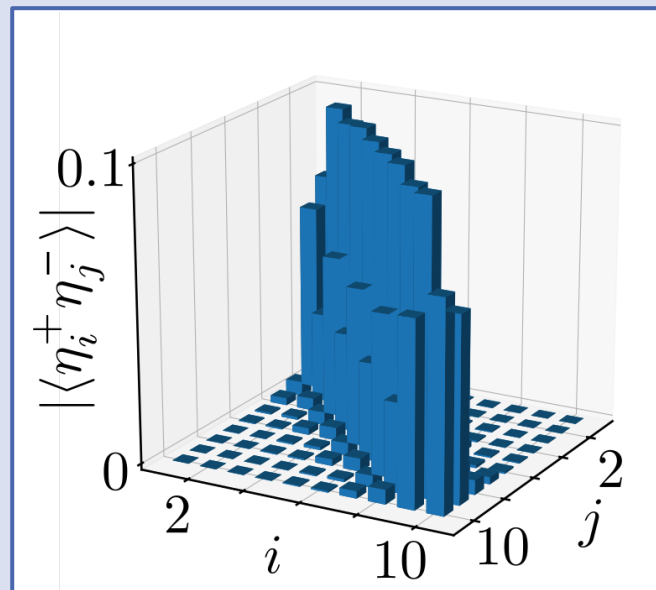
I. Coupling to the Spin Sector

η – correlations in the Hubbard model



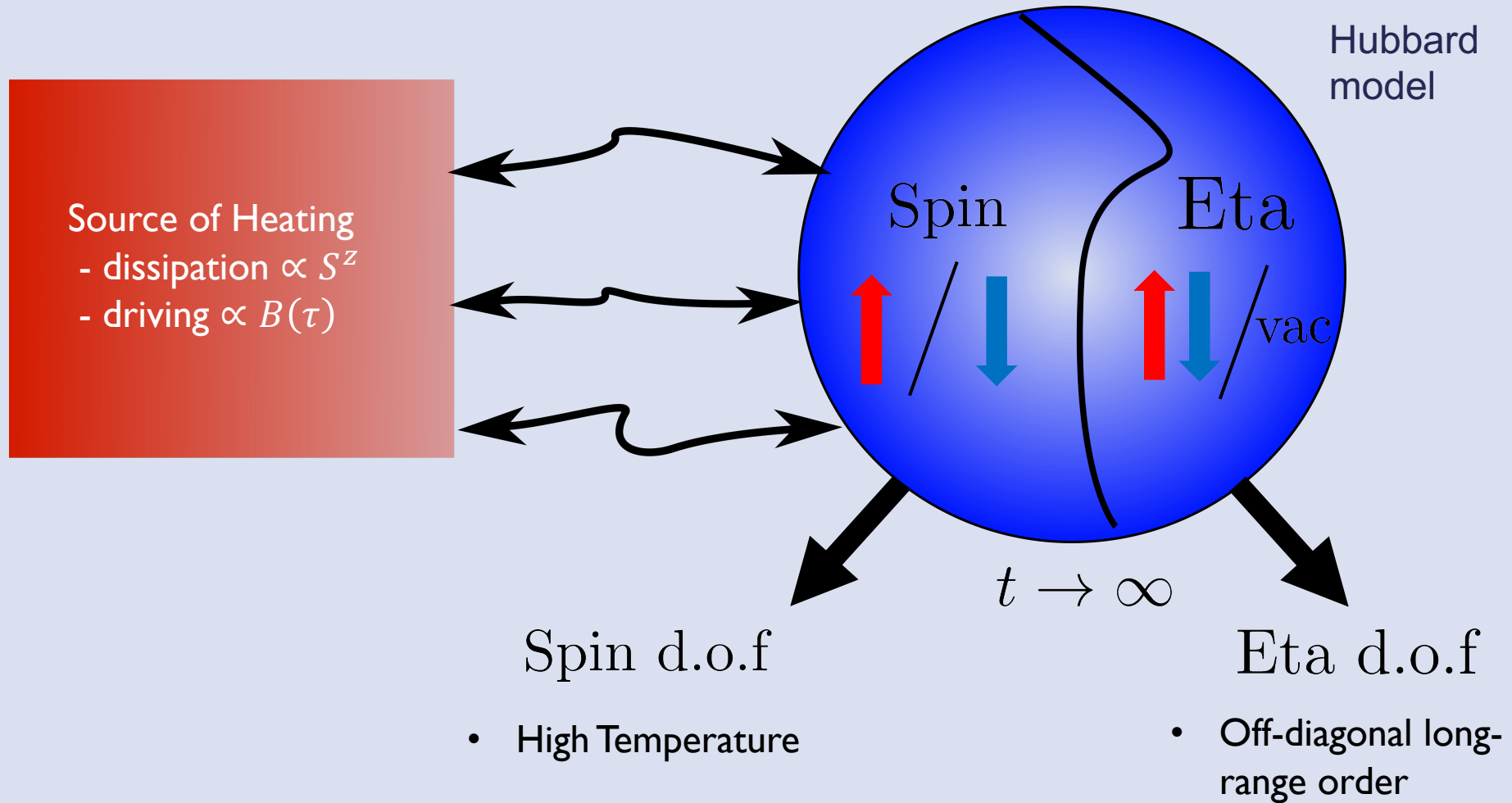
- High energy
- Off-diagonal long-range order
- Superconducting

C. N. Yang, PRL **63**, 2144 (1989)



- **Competing order**
- Decaying correlations

Spin-heated Hubbard model



Bath coupling to the spin sector, $B = 0$



- Conserved quantities are $N_{\downarrow}$, $N_{\uparrow}$, and $\eta^+ \eta^-$

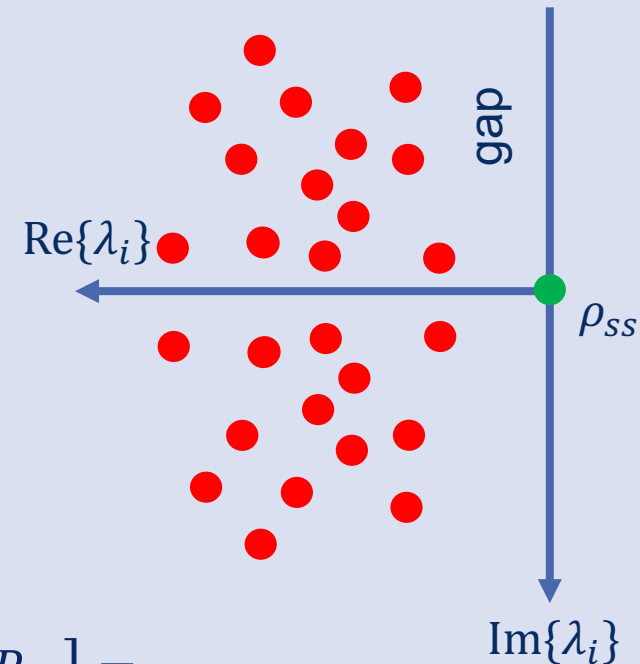
- We assume a stationary state of the form

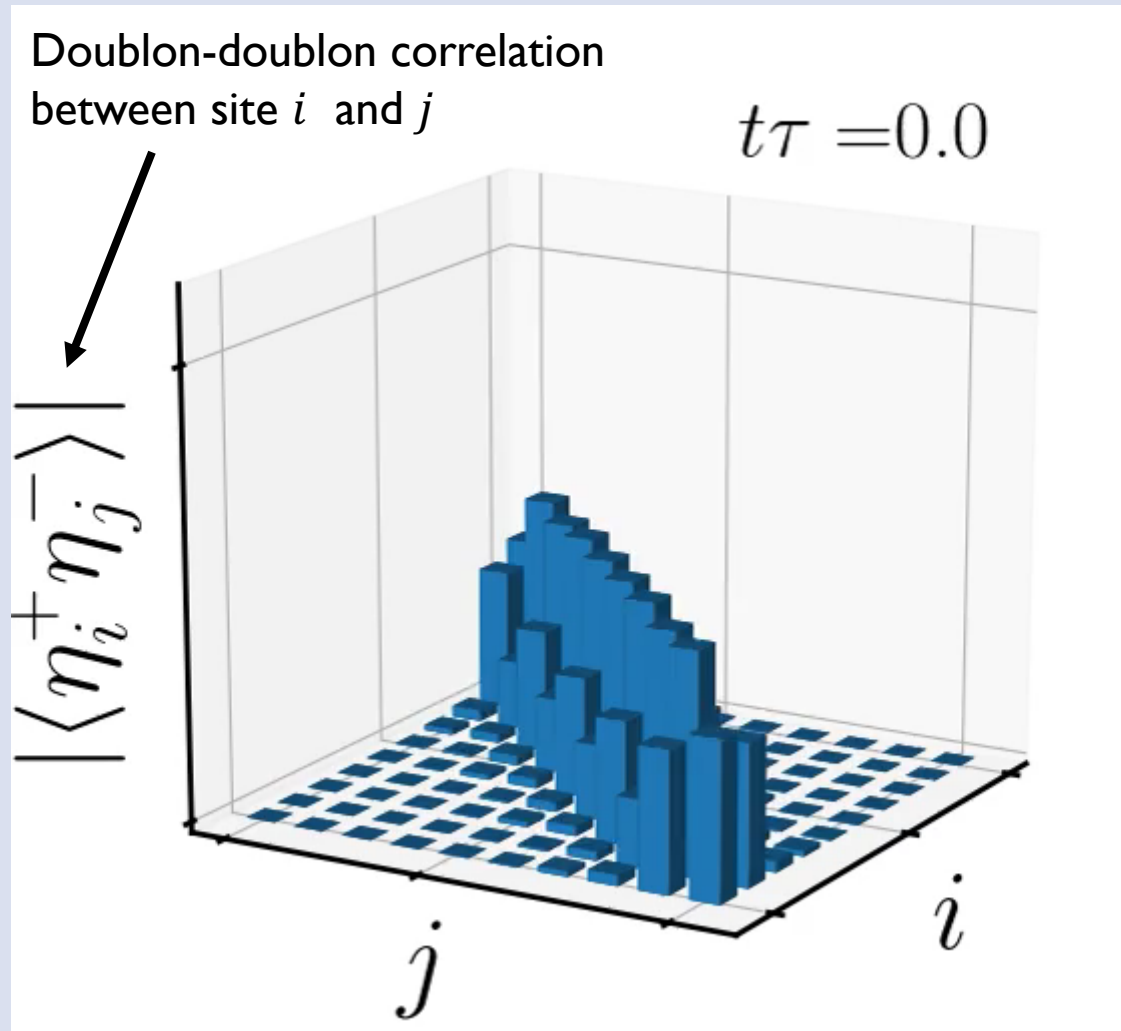
$$\rho_{ss} \propto \exp(\beta_1 N_{\downarrow} + \beta_2 N_{\uparrow} + \beta_3 \eta^+ \eta^-)$$

- Lagrange parameters β_i fixing the conserved quantities.

- Since in the stationary state ρ_{ss} we have $[\rho_{ss}, P_{i,j}] = 0$, where $P_{i,j}$ swaps two lattice sites we can show that

$$\text{Tr}\{\rho_{ss}, \eta_i^+ \eta_{i+j}^-\} = \text{const.}$$





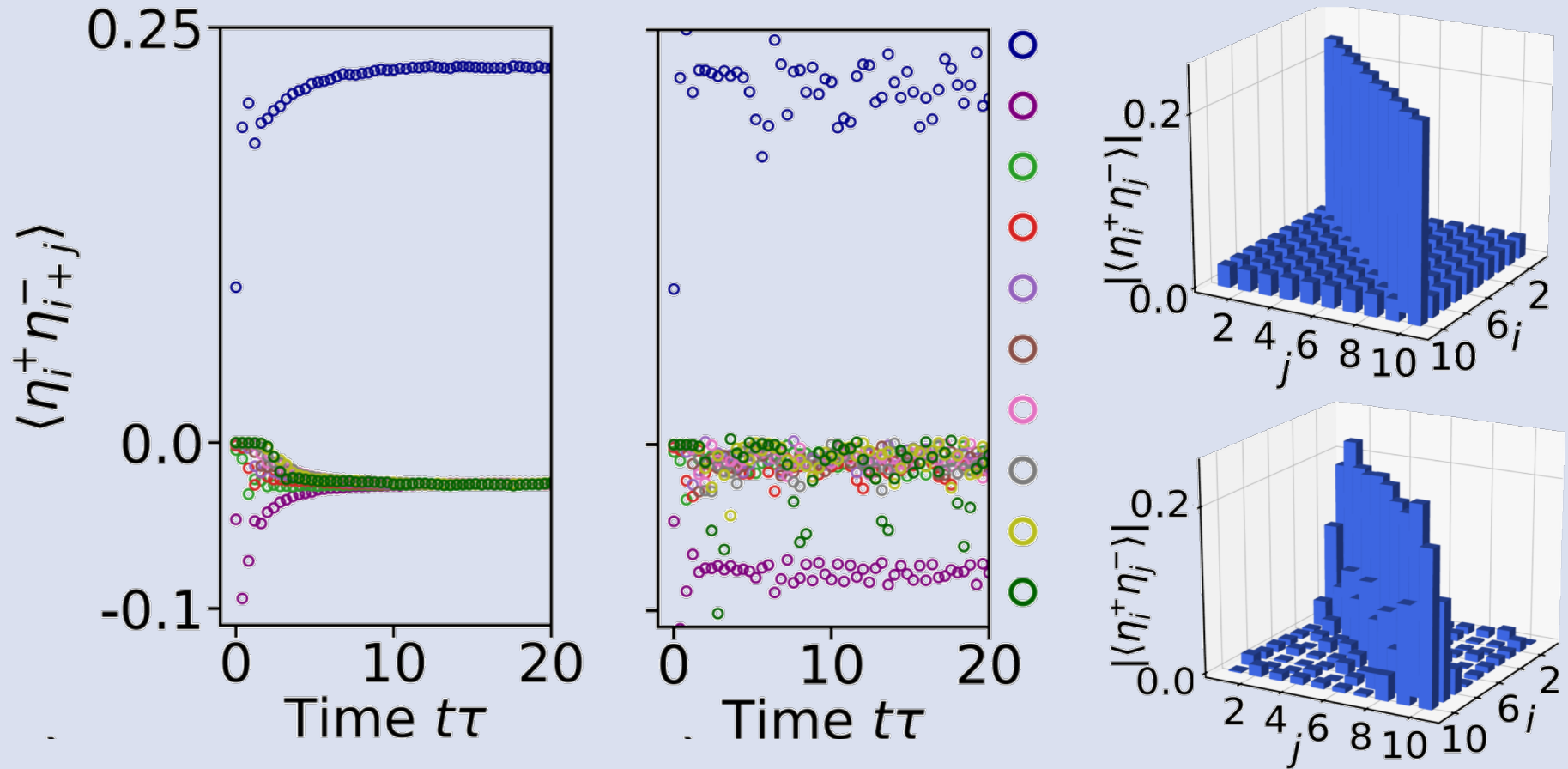
- Uniform off-diagonal Hubbard model
- Dimerizing dimer order
- No off-diagonal order

Parameters:

$$U = t, \gamma = 2t$$

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] + \gamma \sum_j (s_j^z \rho s_j^z - \frac{1}{2} \{(s_j^z)^2, \rho\})$$

A quench with and without dissipation

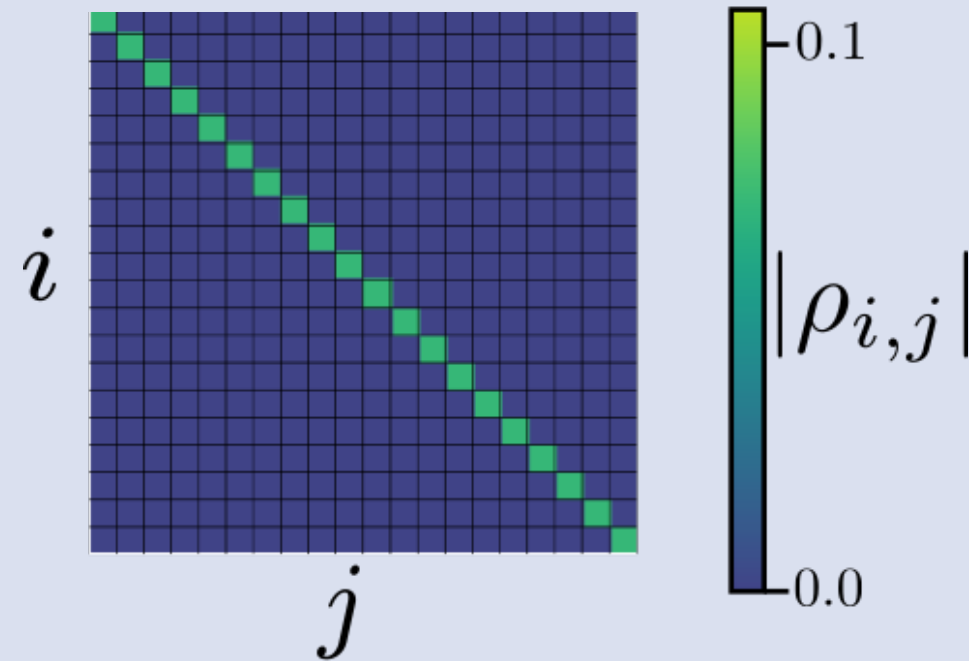


Interaction quench $U = 4\tau \rightarrow U = \tau$ for $\gamma = 0$ and for $\gamma = 2\tau$

Spin- and η -projections

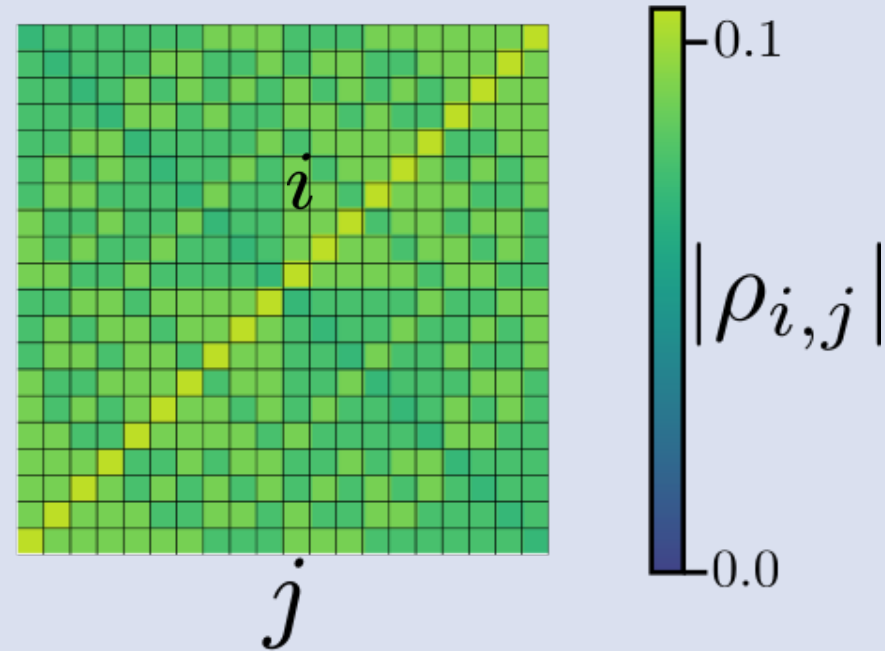
Project into Spin d.o.f.

- Diagonal
- Classical
- Infinite Temperature



Project into Eta d.o.f.

- Off-Diagonal Order
- Entangled
- Low Temperature



$$\frac{\partial \rho}{\partial t} = -i[H, \rho] + \gamma_s D_{\text{wanted}}[\rho] + \gamma_c D_{\text{unwanted}}[\rho]$$

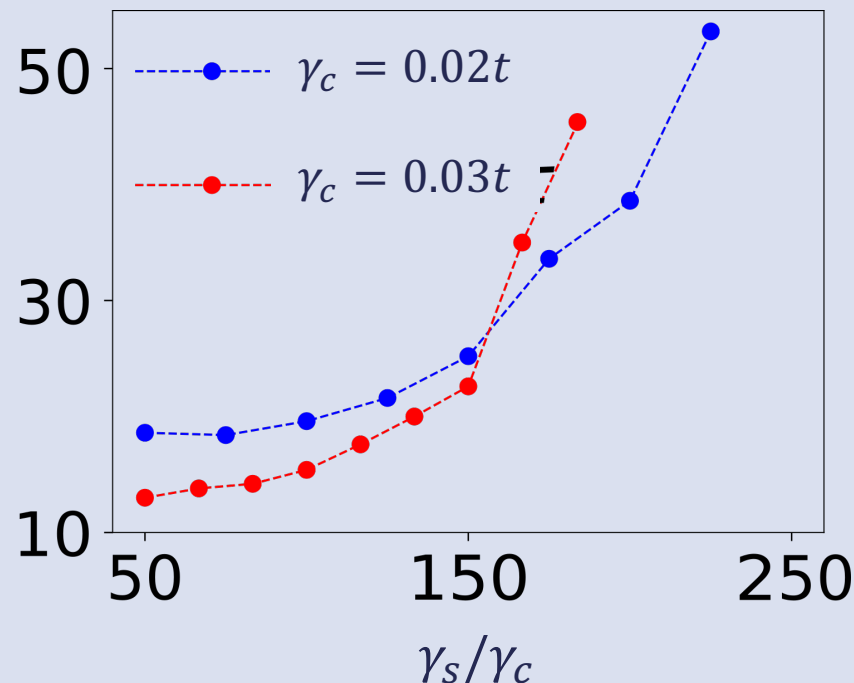
$D_{\text{wanted}}[\rho]$

- Wanted, spin dephasing
- Necessary for η order

$D_{\text{unwanted}}[\rho]$

- Unwanted, charge decoherence
- Destroys η coherences

Time window in which off-diagonal order can be observed $\Delta\tau t$

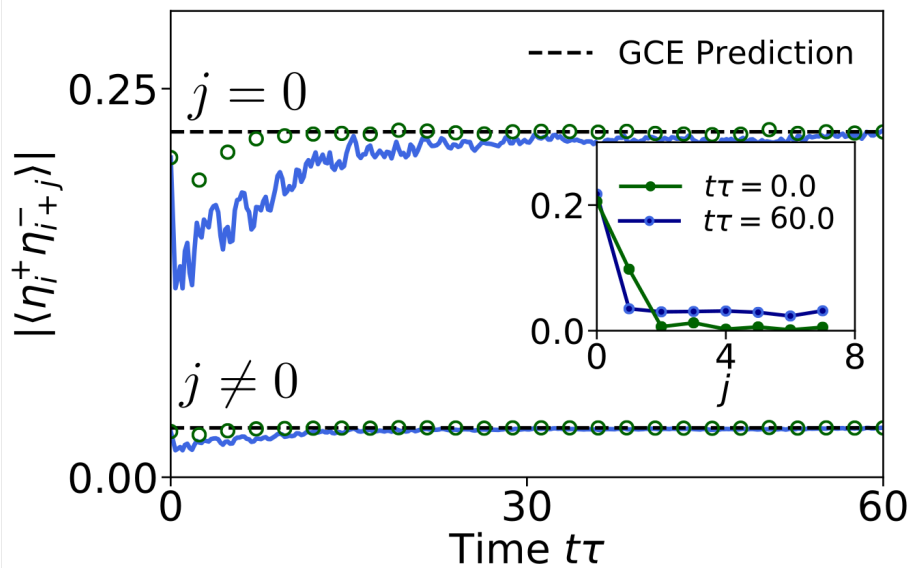
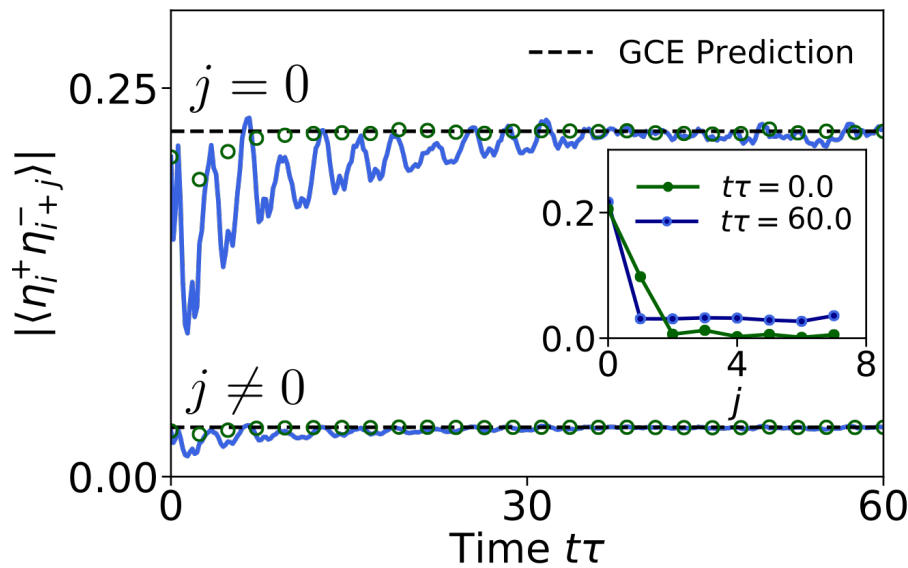


- Wanted dephasing can freeze the unwanted decoherence
- Order appears on an intermediate time-scale

Spin driven Hubbard model $B(\tau)$

- Strongly drive the isolated system in the spin sector with $B(\tau) = V \cos(\Omega\tau)$

$$H_{\text{Hub}} = -t \sum_{\langle i,j \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i + B(\tau) \sum_i f_i S_i^z$$



II. Coupling to the Charge Sector

- We can allow for a trapping potential $\sum_i \epsilon_i n_i$
- The stationary states are given by

$$\rho_{ss} \propto \exp(\beta_0 N + \beta_1 (S^+ S^-) + \beta_2 S^Z)$$

- Initial states that are superpositions of S^Z contain “mixed coherences” of the form (with integer $m, n > 0$)

$$\rho_{nm} = (S^+)^n \rho_{ss} (S^-)^m$$

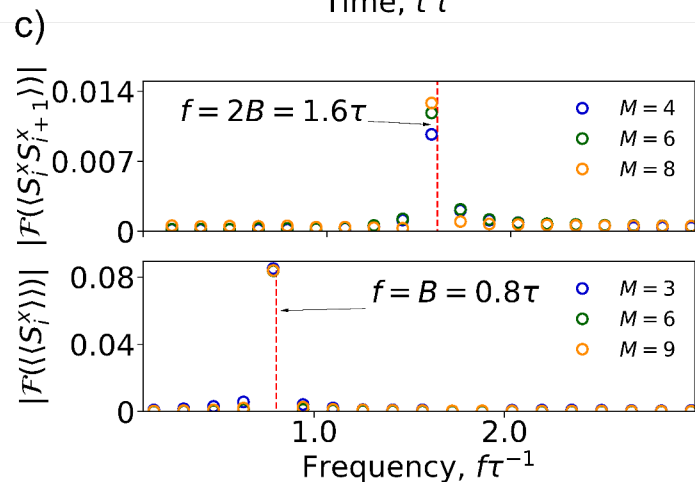
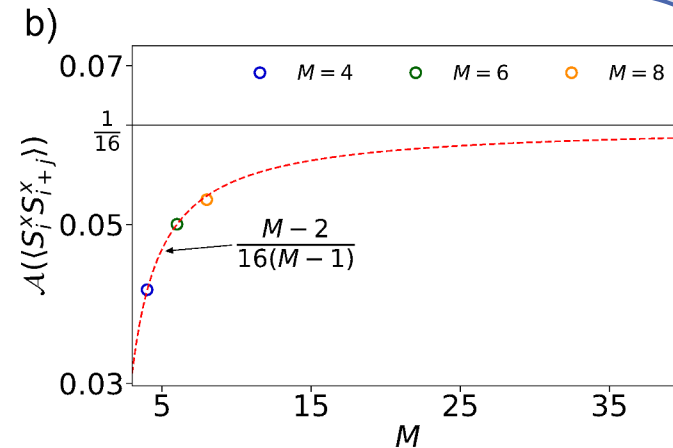
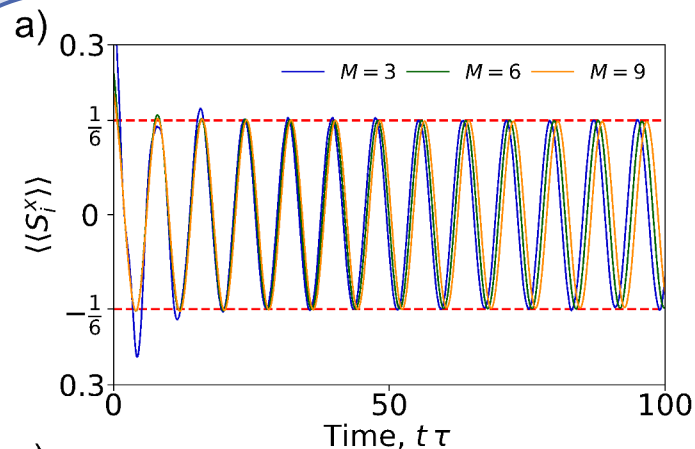
- For $B \neq 0$ these evolve in a non-stationary way according to

$$\mathcal{L}\rho_{nm} = iB(m - n)\rho_{nm}$$

Bulk averaged Hubbard dynamics

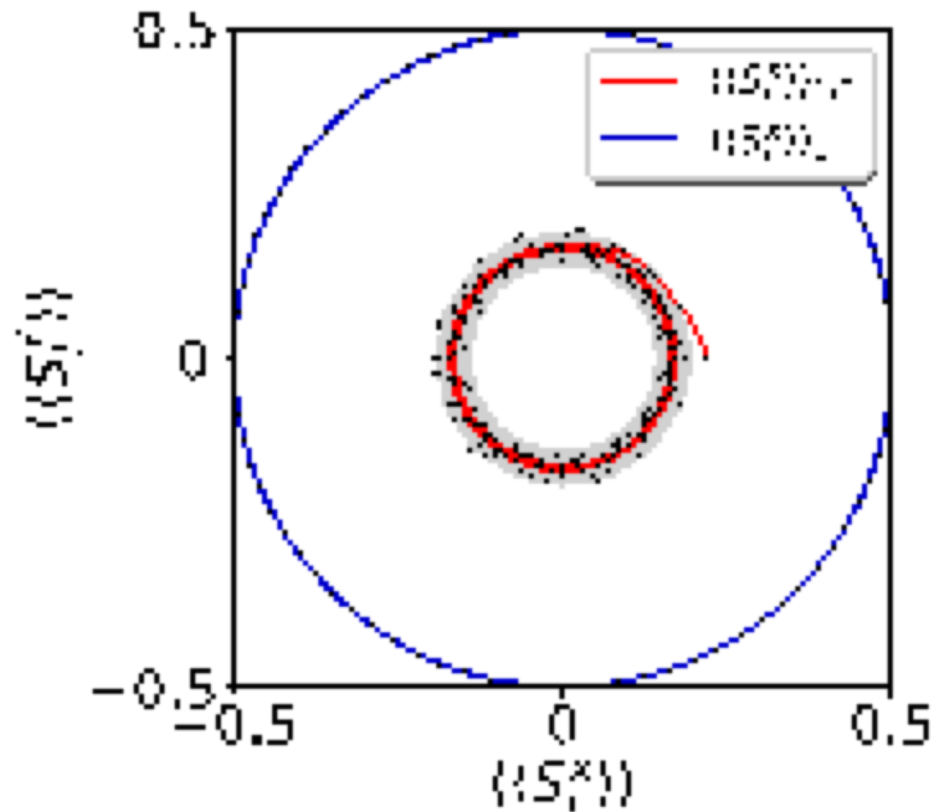
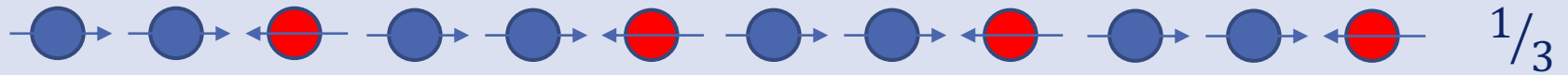


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- Oscillations exactly at multiples of B
- Larger systems require longer to reach oscillatory limit
- Amplitude of oscillations reaches finite value in thermodynamic limit

Quantum trajectory simulation



- Density operator in the long-time limit

$$\lim_{t \rightarrow \infty} \rho(t) = \sum_{n \geq m} C_{n,m} (e^{iB(n-m)t} \rho_{nm} + h.c.),$$

- where the C_{nm} are determined by the initial state
- An observable $X = \prod_n X_n$ that is a product of M operators acting on individual sites will evolve according to

$$\lim_{t \rightarrow \infty} \langle X \rangle(t) = \sum_{n \geq m} D_{nm} \cos(B(m-n)t) + \text{const.}$$

- with $D_{nm} = 2C_{nm} \text{Tr}\{\rho_{nm} X\}$

→ coherent non-decaying limit cycles of X

- We consider perturbation $B \rightarrow B_n = \bar{B} + \delta B_n$ with

$$\epsilon = \frac{\overline{\delta B_n}}{\bar{B}} \ll 1$$

- and expand $\mathcal{L}$ and ρ in powers of ϵ

$$\mathcal{L} = \mathcal{L}^{(0)} + \epsilon \mathcal{L}^{(1)} + \dots$$

$$\rho = \rho^{(0)} + \epsilon \rho^{(1)} + \dots$$

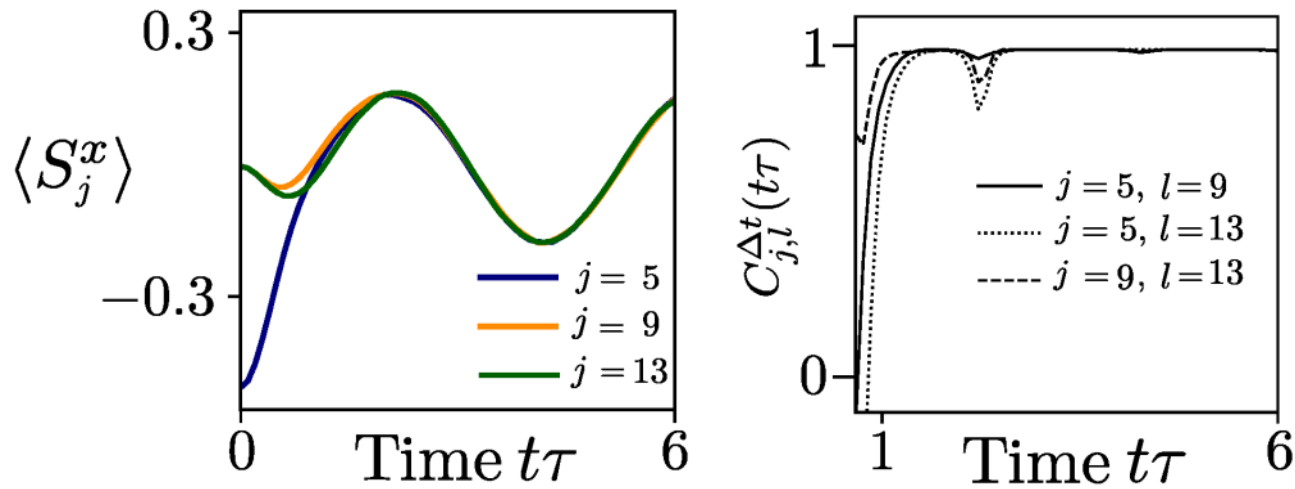
$$\lambda = \lambda^{(0)} + \epsilon \lambda^{(1)} + \dots$$

- For translationally invariant eigenmodes we find

$$\lambda^{(1)} = \text{Tr} \left\{ (\rho^{(0)})^\dagger \mathcal{L}^{(1)} \rho^{(0)} \right\} = 0$$

- and otherwise $\lambda^{(1)}$ is purely imaginary

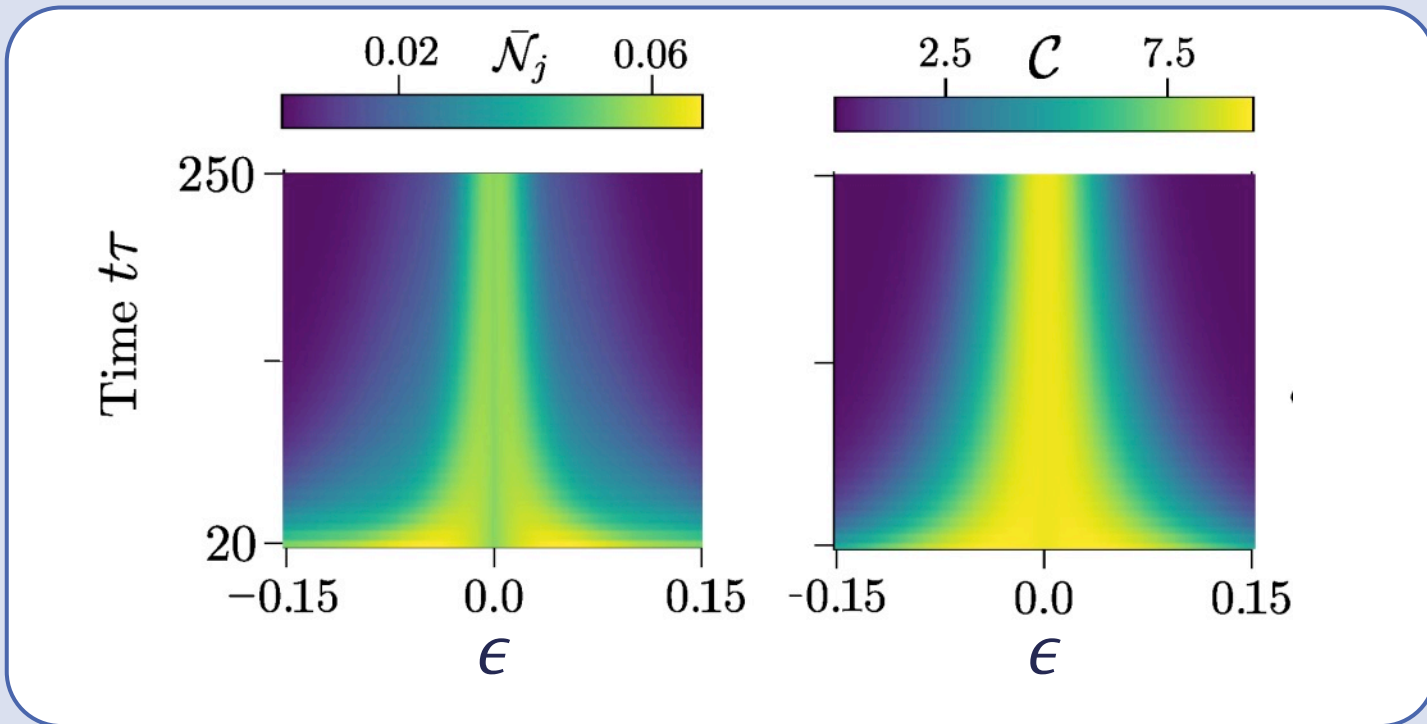
→ limit cycles of X are stable to lowest order



- Observables S_j^x and Pearson correlation coefficient (rolling time window)

$$C_{j,l}^{\Delta t}(\tau) = \frac{\int_{\tau}^{\tau+\Delta t} (S_j^x(\tau') - \bar{S}_j^x)(S_l^x(\tau') - \bar{S}_l^x) d\tau'}{\sqrt{\int_{\tau}^{\tau+\Delta t} (S_j^x(\tau') - \bar{S}_j^x)^2 d\tau' \int_{\tau}^{\tau+\Delta t} (S_l^x(\tau') - \bar{S}_l^x)^2 d\tau'}}$$

with $N = 15$, $U = t$, $\bar{B} = 1.5t$, $\epsilon = 0.1t$.



- Negativity (one site entangled with the others)

$$\mathcal{N}_j(\rho) = \frac{1}{2} (\|\rho^{Tj}\| - 1)$$

- Off-diagonal coherences

$$\mathcal{C} = \sum_{i \neq j} |\rho_{i,j}|$$

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- Orazio Scarlatella
- Paolo Mognini

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