

Cavity-Quantum Enhancement of Superconductivity and Superconducting Polaritons

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SIMONS FOUNDATION

Outline

- ❖ Lessons from the 1970's: Eliashberg mechanism of stimulated superconductivity

- ❖ Quantum cavity-induced Eliashberg effect

- ❖ Superconducting collective modes hybridized with light
 - Superconducting excitons: Bardasis-Schrieffer modes
 - Bardasis-Schrieffer polaritons and s+id condensates
 - Higgs polaritons

Early experimental puzzle (~1970)

- What happens with a superconductor when it is irradiated by electromagnetic field, $E = E_0 e^{i\omega t}$?
- ~~Naïve guess: irradiation should suppress and eventually destroy superconductivity.~~

WRONG!!!



Early experimental puzzle (~1970)

- What happens with a superconductor when it is irradiated by electromagnetic field, $E = E_0 e^{i\omega t}$?
- Contrary to the naïve guess, microwave irradiation enhances superconductivity!
The phenomenon was discovered by Teun Klapwijk et al.

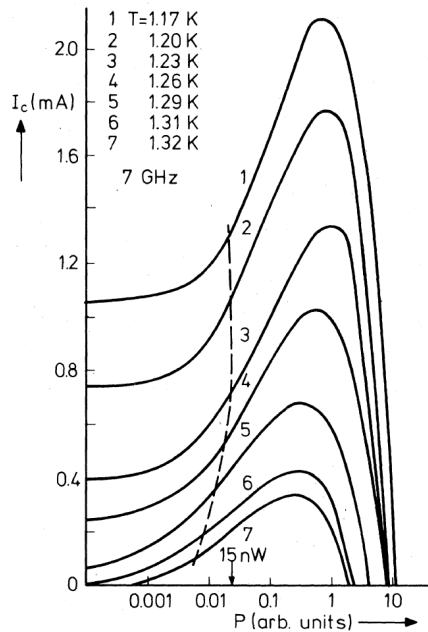


FIG. 2. Critical current I_c as a function of 7 GHz microwave power P with the temperature T of the helium bath

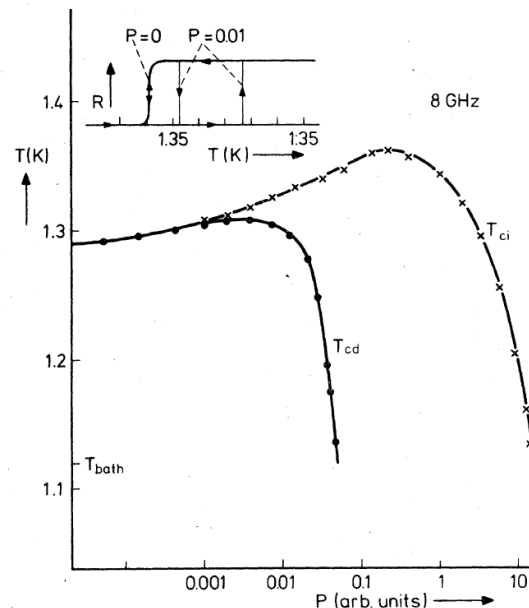


FIG. 12. Measured critical temperatures T_{ci} for increasing temperature and T_{cd} for decreasing temperature as a function of microwave power P . In the inset we have plotted the

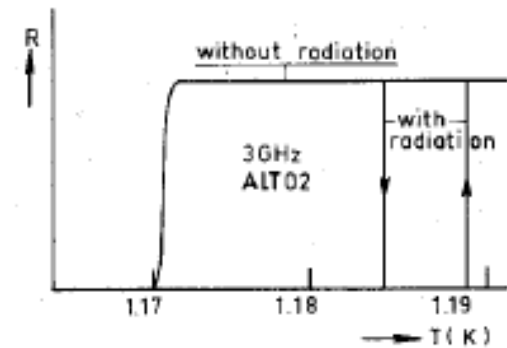


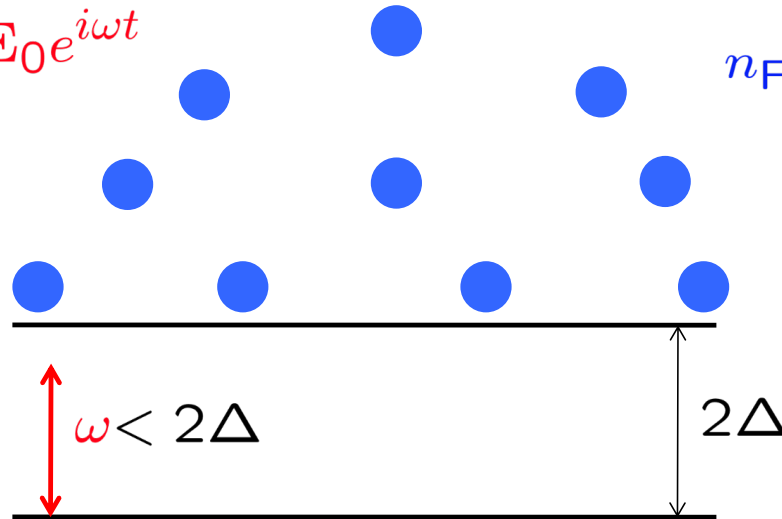
Fig. 5. Transition from the superconducting state to normal and back, with and without radiation (R is the resistance). These curves are measured with a small ac signal (< 1 nA).

Resolution of the puzzle (Gerasim M. Eliashberg)

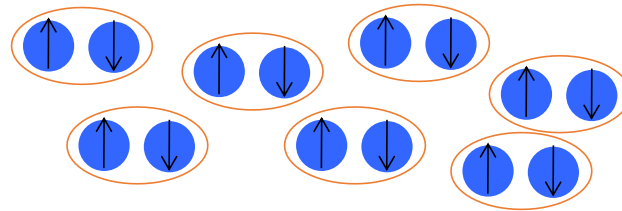
$$E(t) = E_0 e^{i\omega t}$$

$$n_F(\varepsilon) = n_* \left[1 + e^{\frac{\sqrt{\varepsilon^2 + \Delta^2}}{T}} \right]^{-1}$$

γ

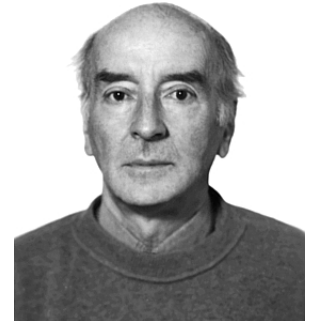


Excitations are “evaporated” up to higher energies, where they settle into a steady state (due to phonons) and do not spoil superconductivity as much.

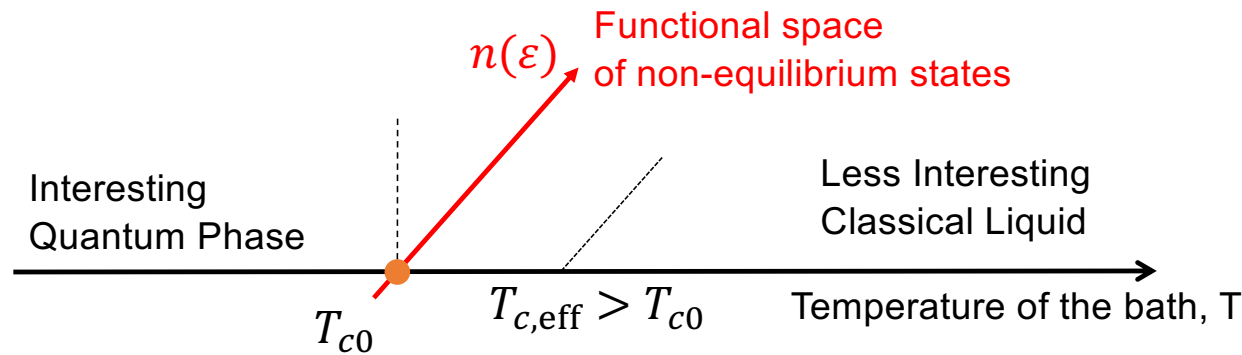


$$1 = U \int d\varepsilon \frac{1 - 2n_F(\varepsilon)}{\sqrt{\varepsilon^2 + \Delta^2}}$$

Annotations: $n_F(\varepsilon) \leftarrow n_*(\varepsilon)$, $\Delta > \Delta_0$



An aside: the Eliashberg stimulation mechanism is generic



- Transition temperature is determined by coupling constant(s) and distribution of excitations (e.g., via a mean-field self-consistency equation).
- In thermal equilibrium, the distribution functions are “non-negotiable.”
- In a generic steady state (possible in the presence of the bath), the order parameter becomes a functional of the distribution function, $\Delta[n(\epsilon)]$.
- Thermal distribution is rarely optimal for the occurrence of a given quantum state. We are interested in states with $|\Delta[n(\epsilon)]| > |\Delta|$ and/or $T_{c,eff} > T_{c0}$.

Two other examples of generalized Eliashberg effect

PRL 106, 165701 (2011)

PHYSICAL REVIEW LETTERS

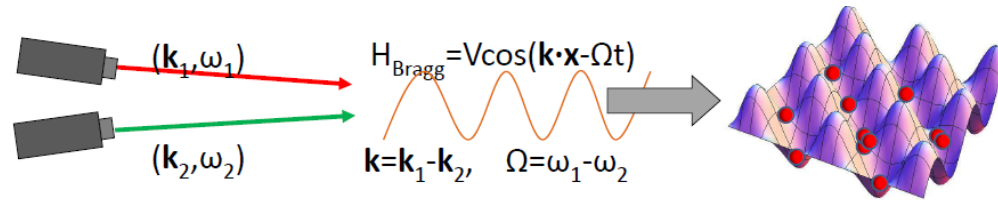
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Dynamic Stimulation of Quantum Coherence in Systems of Lattice Bosons

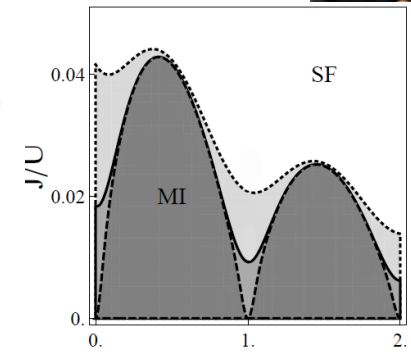
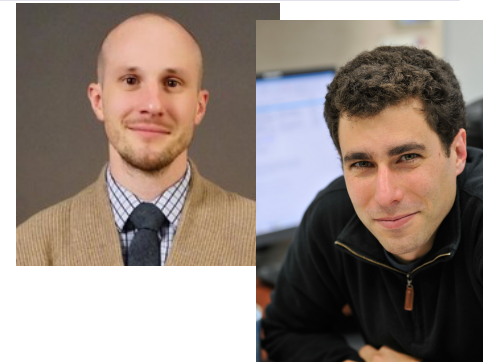
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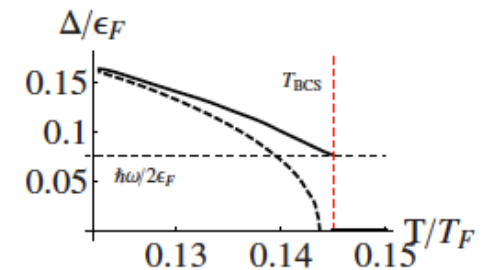
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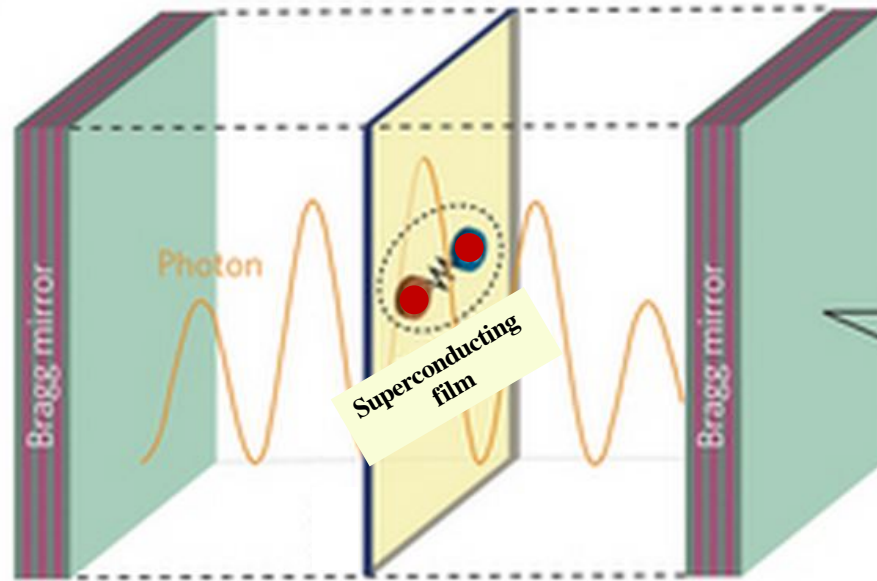
Nonequilibrium enhancement of Cooper pairing in cold fermion systems

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Idea of the 1st part of this talk: replace external irradiation with cavity photons



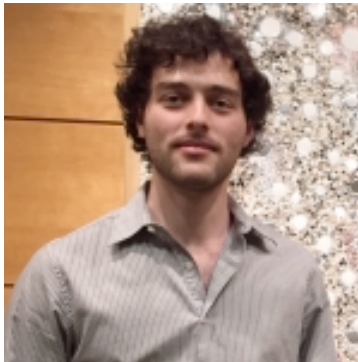
- Use (a lossy) cavity's photons to redistribute electrons into a more favorable distribution (to make the system cooler or “cooler”).
But there are other (perhaps even more interesting) phenomena
- Hybridization of Bogoliubov excitations and photons (strong-coupling limit).
- Coupling photons to collective modes in the superconductor: Higgs, Bardasiss-Schrieffer mode, etc.
- and much more...

Cavity Quantum Eliashberg Enhancement of Superconductivity

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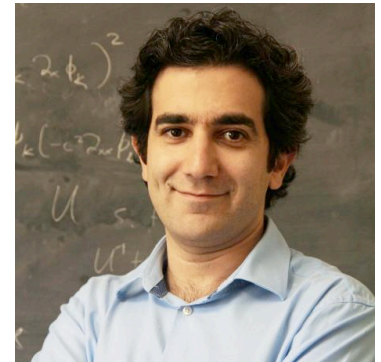
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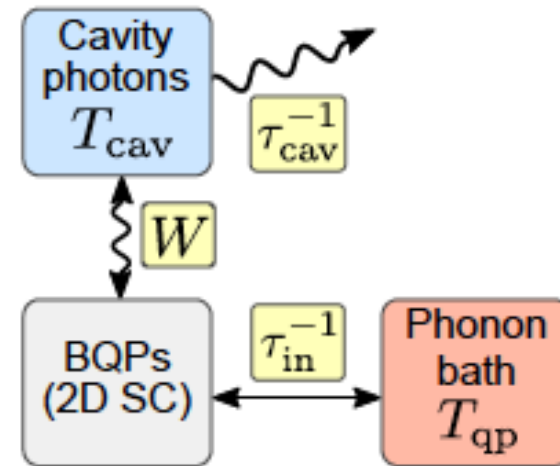


Andrew Allocca
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Mohammad Hafezi

Quantum cavity stimulation of superconductivity: qualitative picture



$$1 = g \int \frac{d\mathbf{p}}{(2\pi)^d} \frac{1 - 2n_{\mathbf{p}}}{\sqrt{\xi_{\mathbf{p}}^2 + \Delta^2}}$$

$$\frac{\partial n}{\partial t} = \mathcal{I}_{\text{cav}}[n] - \frac{n - n_F}{\tau_{\text{in}}}$$

Details of the theory:
1. Coupling to photons

Minimal coupling

$$\hat{H}_{\text{int}} = \int d^2r \hat{\mathbf{J}}(\mathbf{r}, t) \cdot \hat{\mathbf{A}}(\mathbf{r}, t)$$

Electron current

Cavity photon field

$$\hat{\mathbf{A}} = \sum_{\mathbf{q}, \alpha} \sqrt{\frac{2\pi c^2}{\omega_{\mathbf{q}}}} [\hat{a}_{\mathbf{q}, \alpha} \epsilon_{\mathbf{q}, \alpha} + \text{h.c.}] e^{i\mathbf{q} \cdot \mathbf{r}}$$

$$\hat{\mathbf{J}} = e \sum_{\mathbf{k}, \mathbf{k}'} \hat{\psi}_{\mathbf{k}'}^\dagger \left(\frac{\mathbf{k} + \mathbf{k}'}{2m} \right) \hat{\psi}_{\mathbf{k}} e^{i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{r}}$$

Electron creation/annihilation operators

But the excitations in a superconductor are not fundamental electrons but Bogoliubov quasiparticles...

Current in terms of the Bogoliubov quasiparticles

$$\hat{\mathbf{J}} = e \sum_{\mathbf{k}, \mathbf{k}'} \hat{\Psi}_{\mathbf{k}'}^\dagger \left(\frac{\mathbf{k} + \mathbf{k}'}{2m} \right) \hat{\Psi}_{\mathbf{k}} e^{i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{r}}$$

Electrons

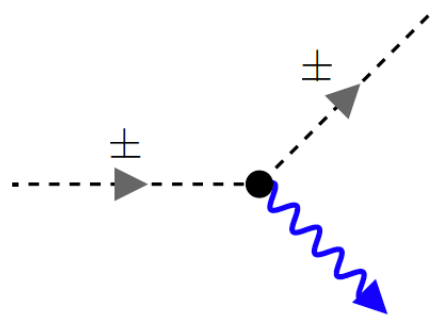
$$\Psi_{\mathbf{k}} = \begin{pmatrix} u_{\mathbf{k}} & -v_{\mathbf{k}} \\ v_{\mathbf{k}} & u_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} \gamma_{\mathbf{k},+} \\ \gamma_{-\mathbf{k},-}^\dagger \end{pmatrix}$$

Quasiparticles

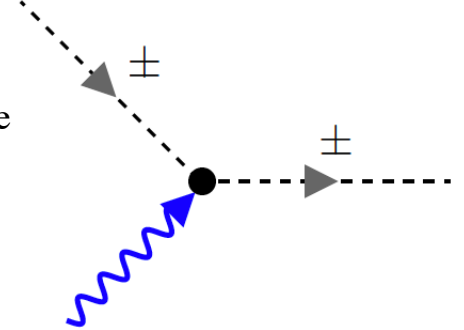
$$\begin{aligned} H^{\text{int}} \sim & \gamma_{\pm}^\dagger \gamma_{\pm} a + \gamma_{+}^\dagger \gamma_{-}^\dagger a + \gamma_{+} \gamma_{-} a \\ & + \gamma_{\pm}^\dagger \gamma_{\pm} a^\dagger + \gamma_{+}^\dagger \gamma_{-}^\dagger a^\dagger + \gamma_{+} \gamma_{-} a^\dagger \end{aligned}$$

Classifying processes in the cavity

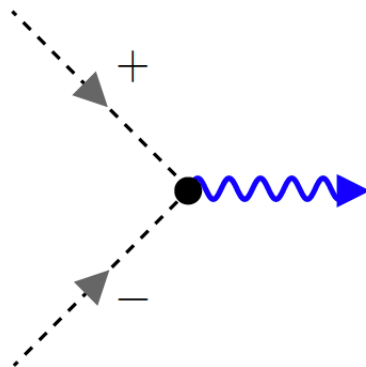
- Relaxation of high-E Bogoliubov excitations



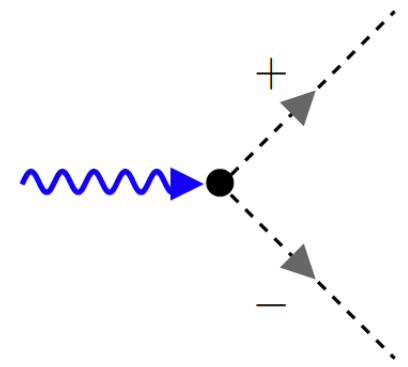
- Scattering process such as absorption of a quantum photon is a QED analogue of the Eliashberg effect



- Recombination of Bogoliubov quasiparticles (cooling effect)



- Pair-breaking (heating effect)



Detailed balance between the two processes is broken if the photon and phonon distributions are different.

Details of the theory:
2. Kinetic equation

Kinetic equation. Collision integral

- Kinetic equation:
$$\frac{\partial n}{\partial t} = \mathcal{I}_{\text{cav}}[n] - \frac{n - n_F}{\tau_{\text{in}}}$$

- Collision integral
$$\mathcal{I}_{\text{cav}}[n] = - \int \frac{d\mathbf{k}'}{(2\pi)^2} W_{\mathbf{k}, -\mathbf{k}'} \left[n_{\mathbf{k}} n_{-\mathbf{k}'} (1 + N(E_{\mathbf{k}} + E_{-\mathbf{k}'})) \right. \\ \left. - (1 - n_{\mathbf{k}})(1 - n_{-\mathbf{k}'}) N(E_{\mathbf{k}} + E_{-\mathbf{k}'}) \right]$$

$\nearrow$ BQP's recombination rate
 $\nwarrow$ BQP's distribution $\searrow$ photon distribution

- The recombination rate follows from Fermi's golden rule:

$$W_{\mathbf{k}, -\mathbf{k}'} = \sum_{\alpha} \int \frac{d^2 q}{(2\pi)^2} |\langle \mathbf{k}, -\mathbf{k}' | \mathbf{j}_{\mathbf{q}} | 0 \rangle \cdot \hat{\epsilon}_{\alpha}(\mathbf{q})|^2 \mathcal{A}_{\mathbf{q}}(E_{\mathbf{k}} + E_{-\mathbf{k}'})$$

$\nearrow$ photon spectral function with $q = k - k'$ due to momentum conservation

Suppression of the effect by kinematic constraints.

- Recombination rate in a clean superconductor is very small

$$W_{\mathbf{k},\mathbf{k}'} \propto \left[1 - \frac{\xi\xi' + \Delta^2}{\sqrt{\xi^2 + \Delta^2}\sqrt{\xi'^2 + \Delta^2}} \right] \mathcal{A}_{\mathbf{k}-\mathbf{k}'}$$

The coherence factor vanishes if two BQPs have the same energies.

The photon structure factor quickly decays if the BQP's are different.

- This catch-22 situation occurs because the momentum conservation laws constrain BQPs and photons, which have very different dispersions.
- A similar effect occurs in exciton-polariton systems, but the constraints are relaxed due to disorder. The same happens here.
- There is no such a thing as a clean superconducting film. We have to include disorder anyway.
- The proper technique to describe the system (non-equilibrium, disordered SC) is Keldysh non-linear sigma-model. I will not explain the details (unless you ask), but will just present the results.
The bottom line: disorder relaxes the kinematic constraints in W and enhances the effect.

Solution of the kinetic equation

- ... is very simple (in a perturbative steady state), once we have all the ingredients

$$\frac{\partial n}{\partial t} = \mathcal{I}_{\text{cav}}[n] - \frac{n - n_F}{\tau_{\text{in}}} = 0$$



$$\delta n(\mathbf{k}) \equiv n(\mathbf{k}) - n_F(\mathbf{k}) = \mathcal{I}_{\text{cal}}[n_F(\mathbf{k})]\tau_{\text{in}}$$

- The collision integral depends on the state of phonons and photons.

$$\mathcal{I}_{\text{cav}}[n] = - \int \frac{d\mathbf{k}'}{(2\pi)^2} W_{\mathbf{k}, -\mathbf{k}'} [n_{\mathbf{k}} n_{-\mathbf{k}'} (1 + N(E_{\mathbf{k}} + E_{-\mathbf{k}'})) \\ - (1 - n_{\mathbf{k}})(1 - n_{-\mathbf{k}'}) N(E_{\mathbf{k}} + E_{-\mathbf{k}'})]$$

- If the phonons and photons are in thermal equilibrium, [...] = 0, and the correction vanishes. But there is a lot of room to break detailed balance (driving, losses, cooling the cavity, etc)

Quantum cavity enhancement of superconductivity (example)

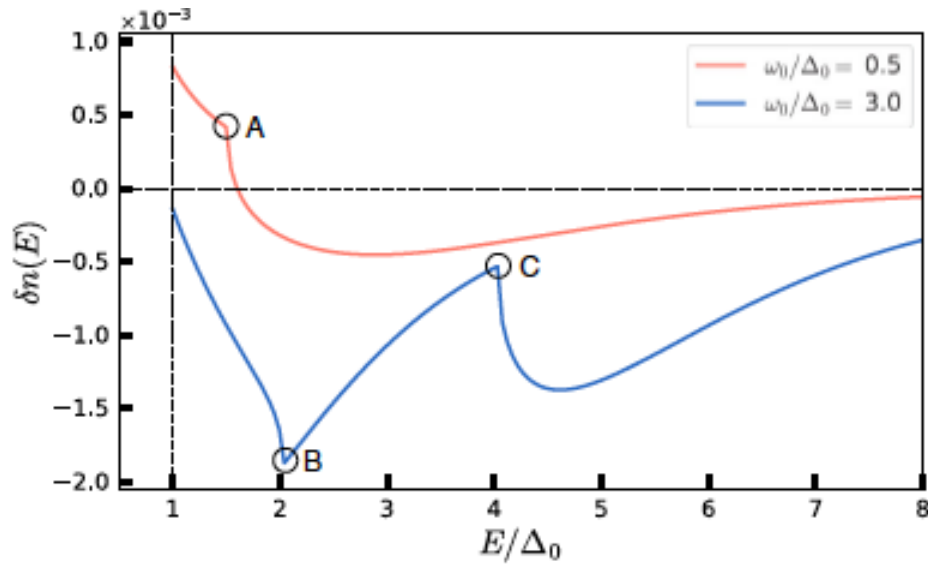


FIG. 2. Change in quasiparticle distribution function due to cavity photons. The two curves are at the same temperature ($T_{\text{cav}}/T_{\text{qp}} = 0.5$) but different cavity frequencies ω_0/Δ_0 . For low cavity frequency (orange), the gap Δ is diminished due to

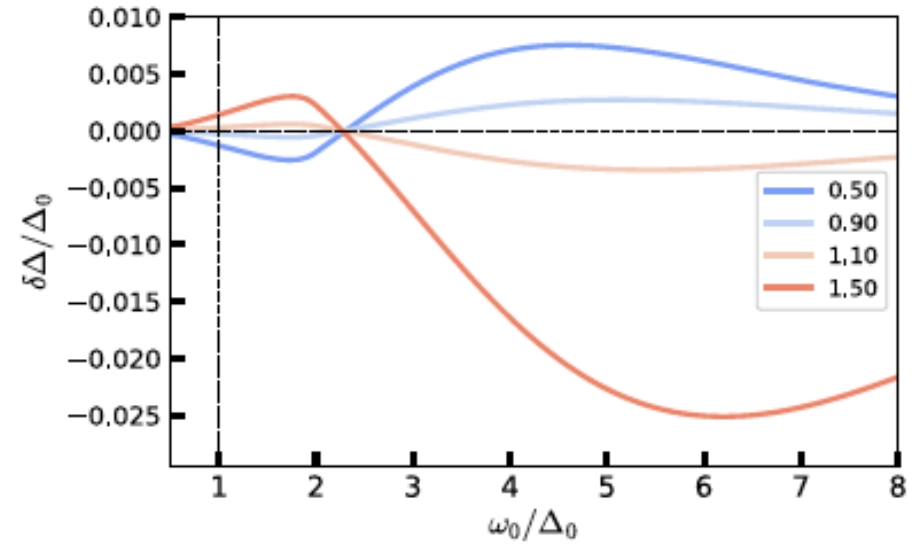


FIG. 3. Gap enhancement $\delta\Delta_0$ for a single-mode cavity, for both cold and hot photons. The y axis is determined by the overall scale $4\pi\alpha D\tau_{\text{in}}T_c^2/((\pi\sqrt{3})^3c^2)X$; with the same values chosen for X and τ_{in} , τ_{cl} , v_F/c as in Fig. 1. Curves are colored and labeled according to the ratio $T_{\text{cav}}/T_{\text{qp}}$, comparing the photon and quasiparticle temperatures. Here, the cavity width is held fixed at $1/2\tau_{\text{cav}} = 10\omega_0$.

*Coherent phenomena in cavity
QED-superconductor systems*

Two-dimensional excitons-polaritons

$$H_{\text{ex}} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \omega_{\text{ex}}(\mathbf{q}) b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}}$$

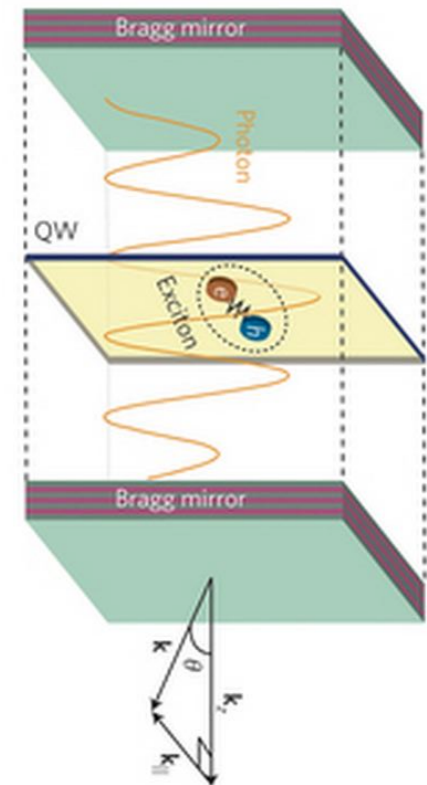
$$H_{\text{phot}} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \omega_{\text{phot}}(\mathbf{q}) a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} \quad \omega_{\text{phot}}(\mathbf{q}) = c^2 \sqrt{q^2 + \left(\frac{n\pi}{L}\right)^2}$$

$$H_{\text{coupling}} = g \int \frac{d^2\mathbf{q}}{(2\pi)^2} b_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + \text{h.c.}$$

$$H = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \begin{pmatrix} b_{\mathbf{q}}^{\dagger} & a_{\mathbf{q}}^{\dagger} \end{pmatrix} \begin{pmatrix} \omega_{\text{ex}} & g \\ g & \omega_{\text{phot}}(\mathbf{q}) \end{pmatrix} \begin{pmatrix} b_{\mathbf{q}} \\ a_{\mathbf{q}} \end{pmatrix}$$

- We have two distinct kinds of degrees of freedom to couple
- Observed BEC, superfluity and other Gross-Pitaevskii physics
- Exciton-exciton interaction is weak, so beyond mean-field physics has not been reached

Carusotto & Ciuti, RMP (2012)



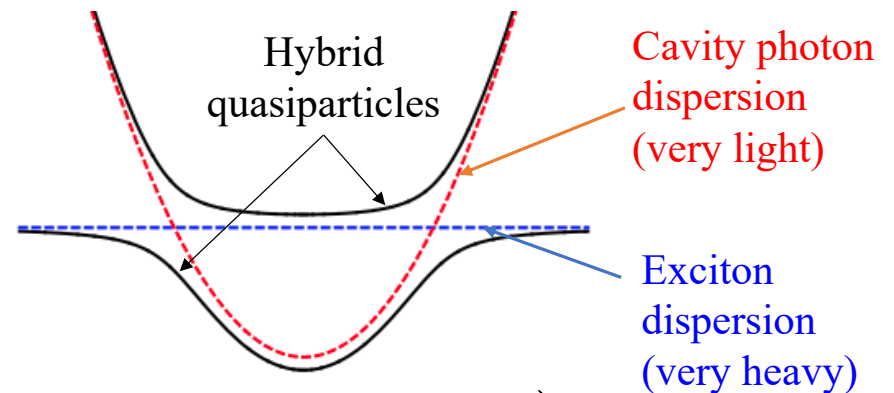
Polariton condensates

- Polaritons are hybrid quasiparticles resulting from the strong coupling between a dipole carrying excitation and light
- In particular, cavity exciton-polaritons have been created and even observed to condense at room temperature



Room-temperature Bose-Einstein condensation of cavity exciton-polaritons in a polymer

Johannes D. Plumhof^{1†}, Thilo Stöferle^{1*}, Lijian Mai¹, Ullrich Scherf² and Rainer F. Mahrt¹



$$H = \begin{pmatrix} \epsilon_{\text{ex}} & g \\ g & \epsilon_{\text{ph}} \end{pmatrix} \Rightarrow E = \frac{1}{2} \left(\epsilon_{\text{ex}} + \epsilon_{\text{ph}} \pm \sqrt{(\epsilon_{\text{ex}} - \epsilon_{\text{ph}})^2 + (2g)^2} \right)$$

- Is it possible to hybridize cavity photons with an excitation of a superconductor?

Cavity superconductor-polaritons

Andrew A. Allocca,^{*} Zachary M. Raines, Jonathan B. Curtis, and Victor M. Galitski

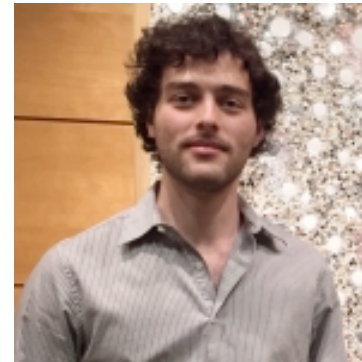
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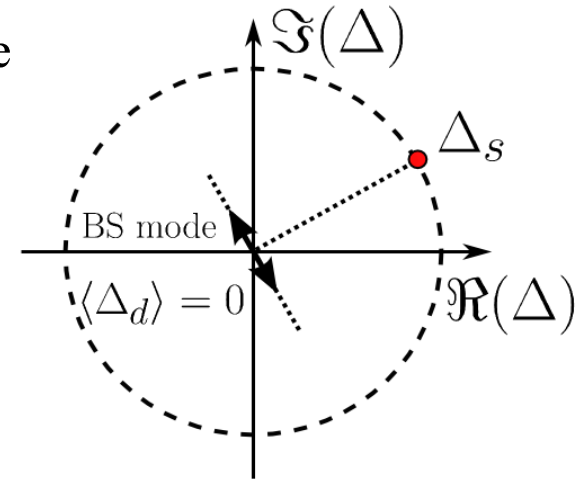
Jonathan Curtis
(UMD->KITP)

Bardasis-Schrieffer modes

- ◆ An analog of excitons inside a superconductor are Bardasis-Schrieffer (BS) modes: Excited modes of Cooper pairs with higher angular momentum relative to the ground state pairing
- ◆ Consider the BCS interaction with sizable s-wave and d-wave channels

$$V(\vec{k}, \vec{k}') = U_s + U_d f_d(\vec{k}) f_d(\vec{k}')$$

- ◆ For $|U_d| < |U_s|$ ground state is an s-wave superconductor, but there is an excitation with d-wave symmetry inside the SC gap.
- The fluctuation that is in-phase with the s-wave component is generally overdamped
- $\langle \Delta_d \rangle = 0$ but $\langle \Delta_d^2 \rangle \neq 0$



Coupling Bardasis-Schrieffer excitons to light

- If Bardasis-Schrieffer modes can be engineered to couple linearly to light, then they could form Bardasis-Schrieffer polaritons
 - Condensation of these modes could produce interesting SC states (s+id)

$$g = \text{---} \Delta_d \text{---} \text{---} \text{---} \vec{A}$$

- Problem: d-wave excitations do not straightforwardly couple in this way (this diagram=0)
 - Solution: The coupling is nonzero when the s-wave condensate has a *gradient*

$$S_{sd} \propto \int d^4x \bar{\Delta}_d(x) (\hat{p}_x^2 - \hat{p}_y^2) \Delta_s(x) + h.c. \quad \Delta_s(x) = \Delta_s e^{i\theta(\vec{r})}$$

- The modes do couple in the presence of disorder and a supercurrent

$$g \sim v_S \Delta_s$$

Bardasis-Schrieffer polaritons

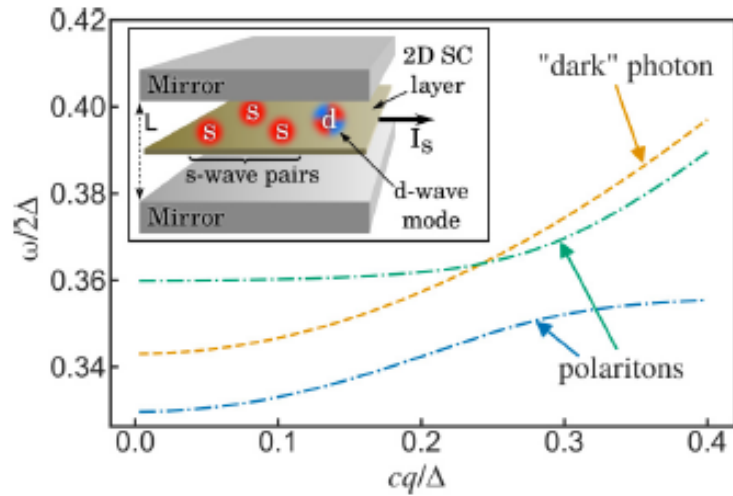


FIG. 1. The dispersion of the Bardasis-Schrieffer-polariton modes (dot-dashed), calculated both numerically and with a simplified analytic method [24]—the two give visually identical results. An external supercurrent causes the BS mode and cavity photons to hybridize, and the polariton states have significant overlap with each. The “dark” photon mode (dashed) remains decoupled. The splitting of otherwise degenerate photon modes is a result of a supercurrent-induced self-energy contribution. Temperature and supercurrent angle are chosen to maximize hybridization (see Fig. 2).

- Driving Bardasis-Schrieffer polaritons into a condensed state would lead to an interesting s+id superconductor.

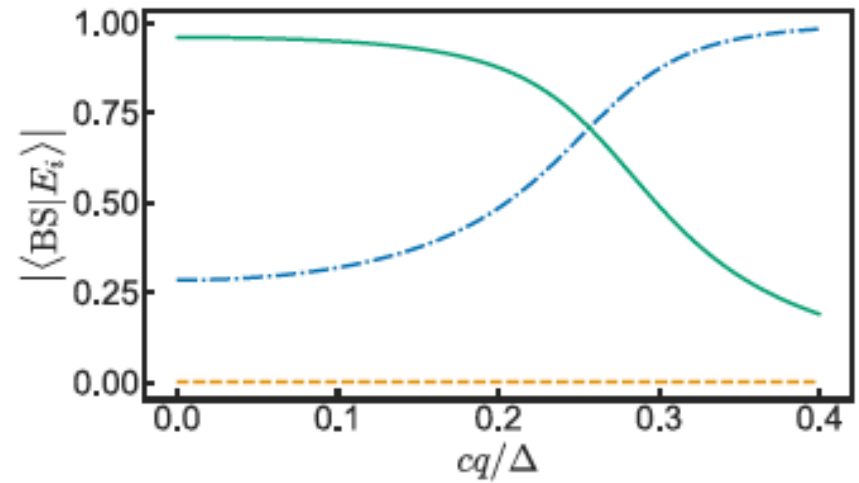


FIG. 3. The Bardasis-Schrieffer component of the eigenvectors of the effective Hamiltonian, Eq. (15) [24]. The upper (solid) and lower (dot-dashed) polaritons have significant photon and Bardasis-Schrieffer character, indicating strong hybridization between the systems. One can also clearly see the “dark” photon mode (dashed) which does not hybridize with the superconductor’s collective mode.

Higgs mode in a superconductor

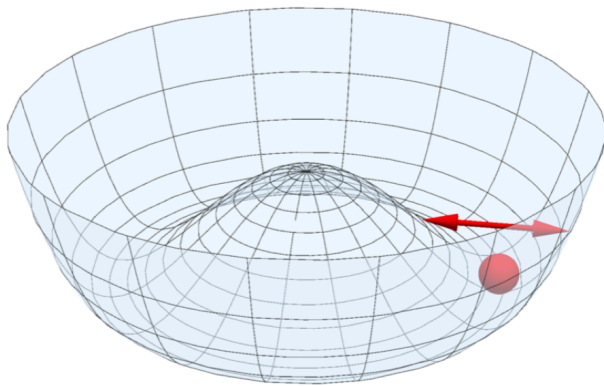
arXiv.org > cond-mat > arXiv:1905.03377

Condensed Matter > Superconductivity

Cavity Higgs-Polaritons

Zachary M. Raines, Andrew A. Allocca, Mohammad Hafezi, Victor M. Galitski

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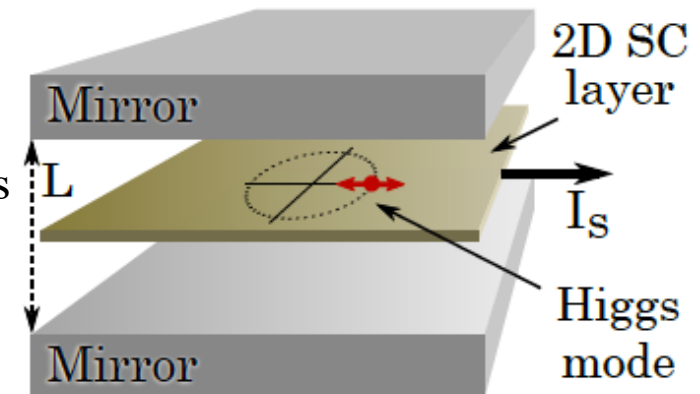


$$\Delta(q, \Omega)e^{i\theta} = (\Delta_0 + \delta\Delta(q, \Omega))e^{i\theta}$$

$$\Omega_{\text{Higgs}} = 2\Delta_0$$

- Higgs is sitting at the edge of quasiparticle continuum and this yields an unusual damping & (Volkov-Kogan) ringdown of the Higgs:

$$\delta\Delta(t) \propto \frac{\cos(2\Delta_0 t)}{\sqrt{t}}$$

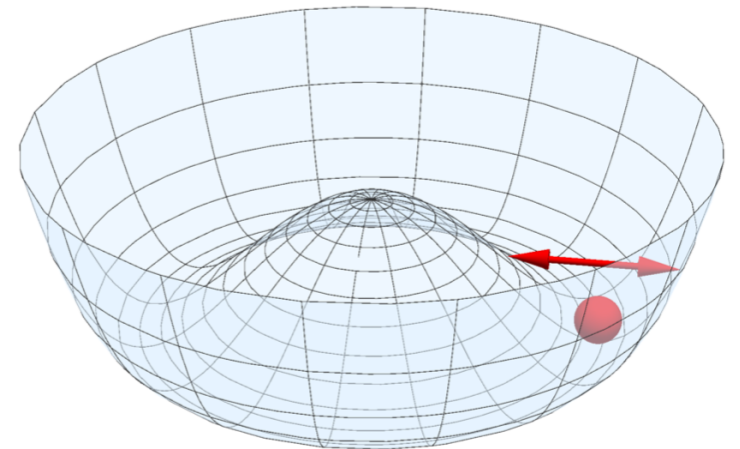


Coupling Higgs modes to light

$$\Delta(q, \Omega)e^{i\theta} = (\Delta_0 + \delta\Delta(q, \Omega))e^{i\theta}$$

$$\Omega_{\text{Higgs}} = 2\Delta_0$$

- Higgs mode $\rightarrow$ amplitude mode of superconductor
- No charge or dipole moment $\rightarrow$ no linear coupling to light
- However, disorder + supercurrent generates a linear coupling



Higgs polaritons

- Higgs mode has no charge or dipole moment (no coupling to light)
- However, disorder & supercurrent generate a linear coupling (Moor, Volkov, & Efetov, PRL 2017)

- The true excitations are linear combinations of photon and Higgs

$$G_R^{-1}(\omega) = \begin{pmatrix} G_h(\omega)^{-1} & g(\omega) \\ g(\omega) & G_A^{-1}(\omega) \end{pmatrix}$$

$$-2\pi A(\omega, \mathbf{q}) = \text{tr} \left(G_R(\omega) - G_A(\omega) \right)$$

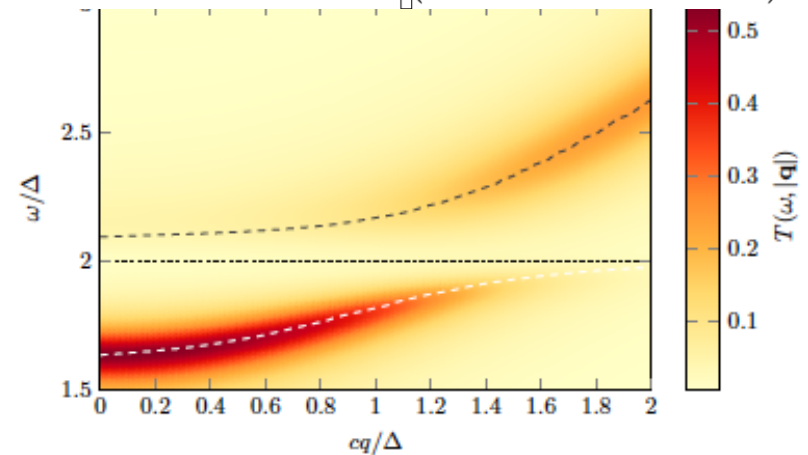


FIG. 4. (Color online) Transmission probability through the cavity as a function of photon frequency ω and momentum transverse to the cavity plane q . The maximum spectral weight for the polariton branches is shown as dashed lines, and the pair-breaking energy 2Δ is plotted as a dotted line.

- Interesting question: what would condensing Higgs polaritons imply?

Summary and outlook

□ Summary of our cavity work so far

- Superconductivity can be enhanced and bootstrapped to higher temperatures by coupling thin-film superconductors to cavity photons
 - Challenges: optimal protocols for driving the cavity are not clear, the effect of superconductor itself on the cavity needs to be considered
- General idea: exciton modes in conventional cavity polaritonic setups can be replaced by other modes (e.g., collective modes in interacting electron systems, including superconductors).
- So far we showed that Bardasis-Schrieffer polaritons and Higgs polaritons can be created. But it is clear that polaritons involving other types of collective modes are possible.

□ Focus of our current and future work: cavity physics meets many-body physics & strong correlations

- Spin physics in Hubbard exciton-polaritons
- Photon-enabled Peierls instability (discussions with Ben Lev and Jonathan Keeling)
- Synthetic gauge fields and possible spin-liquid-like states (in the framework of new Simons collaboration “Ultra-Quantum Matter”)
- Strongly-interacting exciton-polariton condensates in topological Kondo insulators
- Creating and manipulating topological superconductivity in cavity-superconductor systems
- Condensing the Higgs-polaritons – connections to t-dependent BCS problem of Yuzbashyan-Altshuler

Our Theory Group



Hilary Hurst, Efim Rozenbaum, Sergey Syzryanov, Aydin Keser, Valentin Stanev, Zach Raines, Andrew Allocca, Mehdi Kargarian, Victor Galitski, Yi-Hua Lai, Jon Curtis

Thank you!