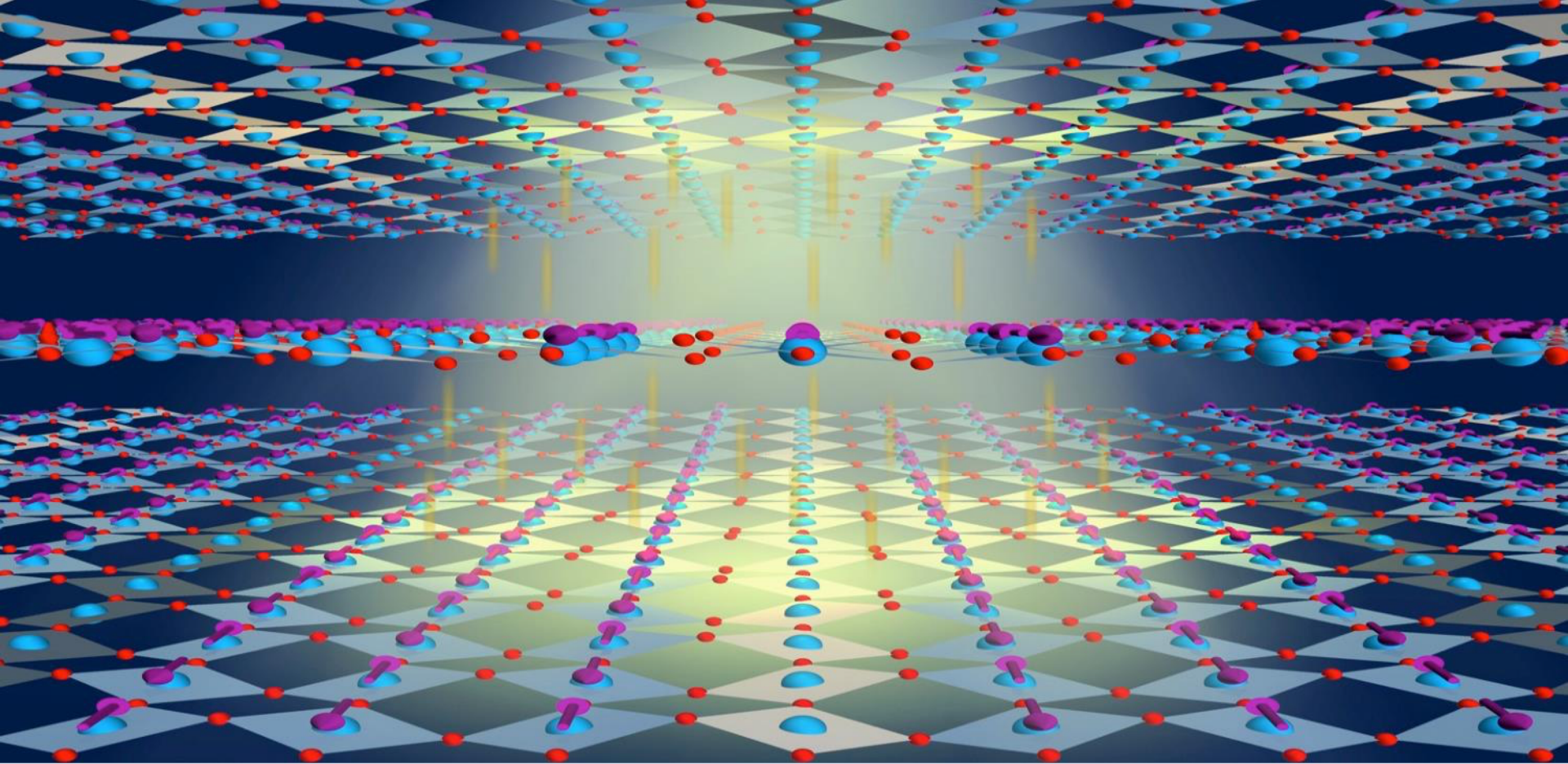


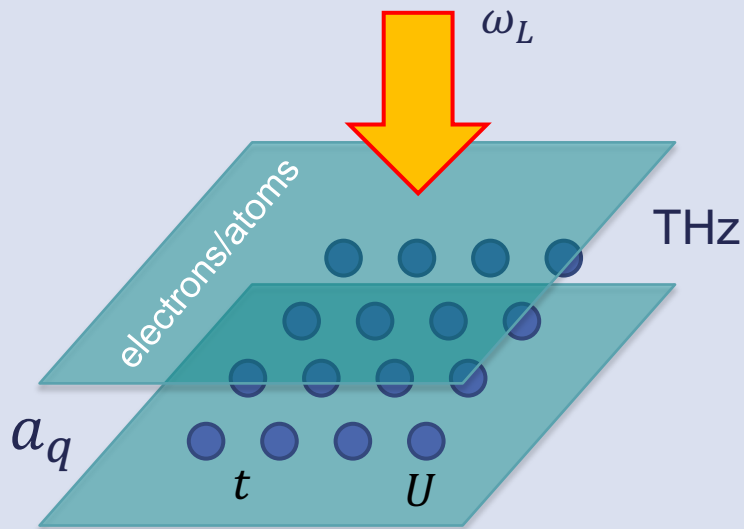


Quantum matter in cavities

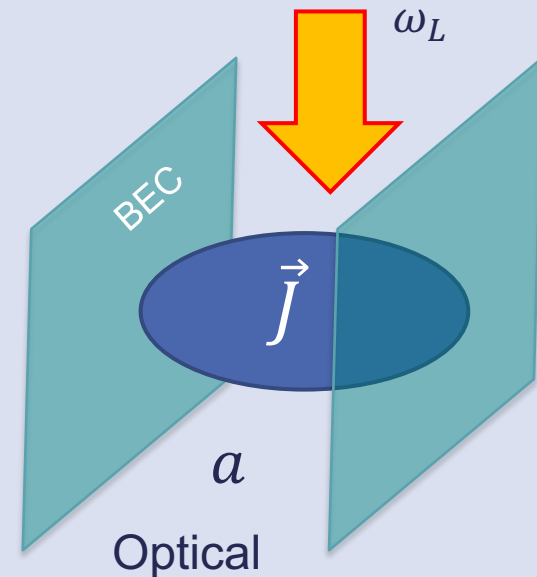


Dieter Jaksch, University of Oxford

9 July 2019

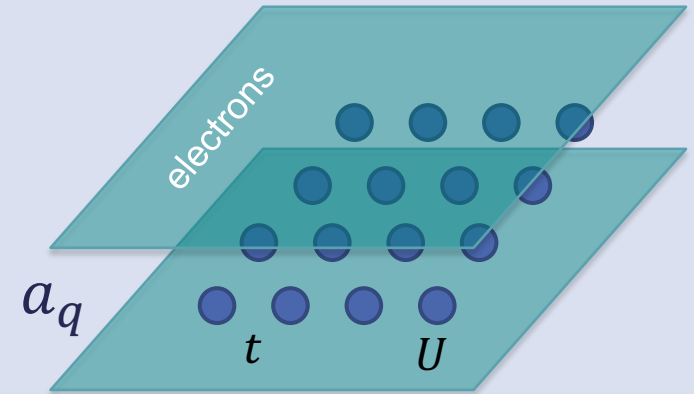


- Cavity-mediated electron-photon superconductivity
- Topological phases of matter
- Driven electron pairing



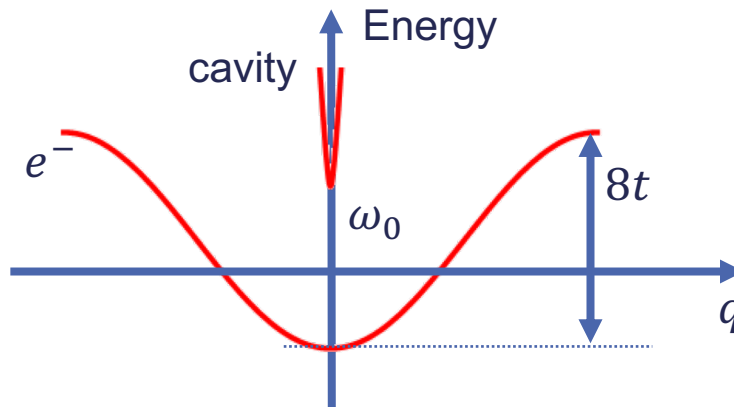
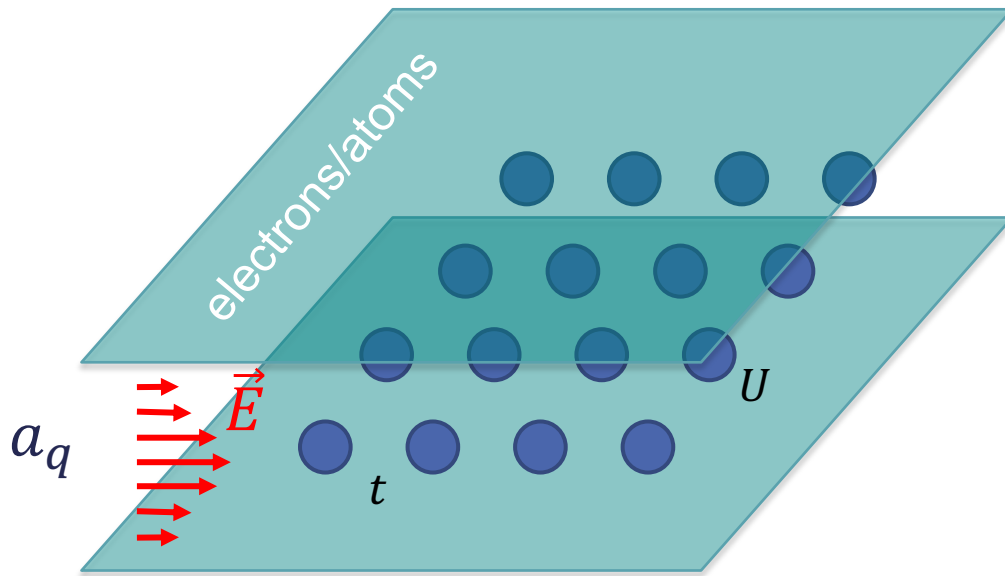
- Non-stationary dynamics
- Quantum squeezing and entanglement
- Beyond mean-field dynamics

- Investigations in free space
- T. Holstein et al., PRB 8, 2649 (1973)
- M Yu. Reizer, PRB 39, 1602 (1985)
- D.V. Kveshchenkov, PRB 47, 3446 (1993)
- D.V. Kveshchenkov, et al., PRB 48, 10766 (1993)
- ...
 - Effects only visible at unrealistically low temperatures



- Cavity mediated interactions
- THz cavity → virtual photon exchange
- Small mode volume → strong coupling
- Photonic band gap → suppression of low frequency photon exchange
- Reduced group velocity for small momenta → better match to Fermi velocity

A 2D electron gas in a cavity at $U = 0$



Small mode volume

$$V \approx 10^{-5} \lambda^3.$$

Free 2D lattice electron gas

$$H = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j$$

coupled by **current operator** $\mathfrak{J}$

$$H_{\text{int}} = g(a_q^\dagger + a_q)\mathfrak{J}$$

To the cavity modes a_q .

The basic cavity frequency ω_0 is chosen such that the electron and cavity bands do not cross

- Vector potential for lowest mode in z-direction

$$\vec{A}' = \sum_{\vec{q}} \sqrt{\frac{\hbar}{\epsilon\epsilon_0 V \omega_q}} e^{i(q_x x + q_y y)} \begin{pmatrix} (b_x - b_x^\dagger) i \sin(\frac{\pi z}{L}) \\ (b_y - b_y^\dagger) i \sin(\frac{\pi z}{L}) \\ (b_z + b_z^\dagger) \cos(\frac{\pi z}{L}) \end{pmatrix}$$

- Boundary conditions and transversality $\vec{q} \cdot \vec{A} = 0$

$$\vec{A}(\vec{r}) = i \sum_{\vec{q}, s} \sqrt{\frac{\hbar}{\epsilon\epsilon_0 V \omega_q}} e^{i(q_x x + q_y y)} \hat{e}_{\vec{q}, s} (a_{\vec{q}, s} + a_{-\vec{q}, s}^\dagger)$$

$$\hat{e}_{\vec{q}, 1} = \frac{\vec{q}}{|\vec{q}|}$$

$$\hat{e}_{\vec{q}, 2} = \hat{e}_{\vec{q}, 1} \times \hat{e}_z$$

- The Hamiltonian for the cavity thus reads

$$H_f = \sum_q \hbar \omega_q a_q^\dagger a_q$$

with dispersion $\omega_q = \omega_0 \left(1 + \left(\frac{cq}{\omega_0}\right)^2\right)^{\frac{1}{2}}$ and gap ω_0 .

The fermions are described by a simple non-interacting lattice gas on N sites

$$H_0 = \sum_{k,\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma}$$

The coupling to the cavity to the current operator is

$$H_{\text{int}} = \sum_{k\sigma q\epsilon} \frac{g_{k\epsilon}^{(q)}}{\sqrt{N}} (a_{q\epsilon} + a_{-q\epsilon}^\dagger) c_{k+q\sigma}^\dagger c_{k\sigma}$$

where we the cavity coupling strengths are $\propto 1 / V$

We integrate out the cavity using $\hbar\omega_0 \gg |\epsilon_k - \epsilon'_k|$ to obtain an effective interaction

$$V_{\text{eff}} = \sum_{k\sigma k'\sigma'} \sum_{q\epsilon} \frac{g_{k\epsilon}^{(q)} g_{k'\epsilon}^{(-q)}}{\hbar\omega_q} c_{k+q\sigma}^\dagger c_{k\sigma} c_{k'-q\sigma'}^\dagger c_{k'\sigma'}$$

- We find for the interaction potential between electrons with momenta

$$\vec{k} = \vec{Q} + \vec{p} \text{ and } \vec{k}' = \vec{Q} - \vec{p}'$$

$$V_{\vec{p}, \vec{p}'}^{(\vec{Q})} = -\frac{V_0}{2} \frac{\sum_{j=x,y} \cos[(p_j + p'_j)a] - \cos(2Q_j a)}{(\omega_0/c)^2 + |\vec{p} - \vec{p}'|^2}$$

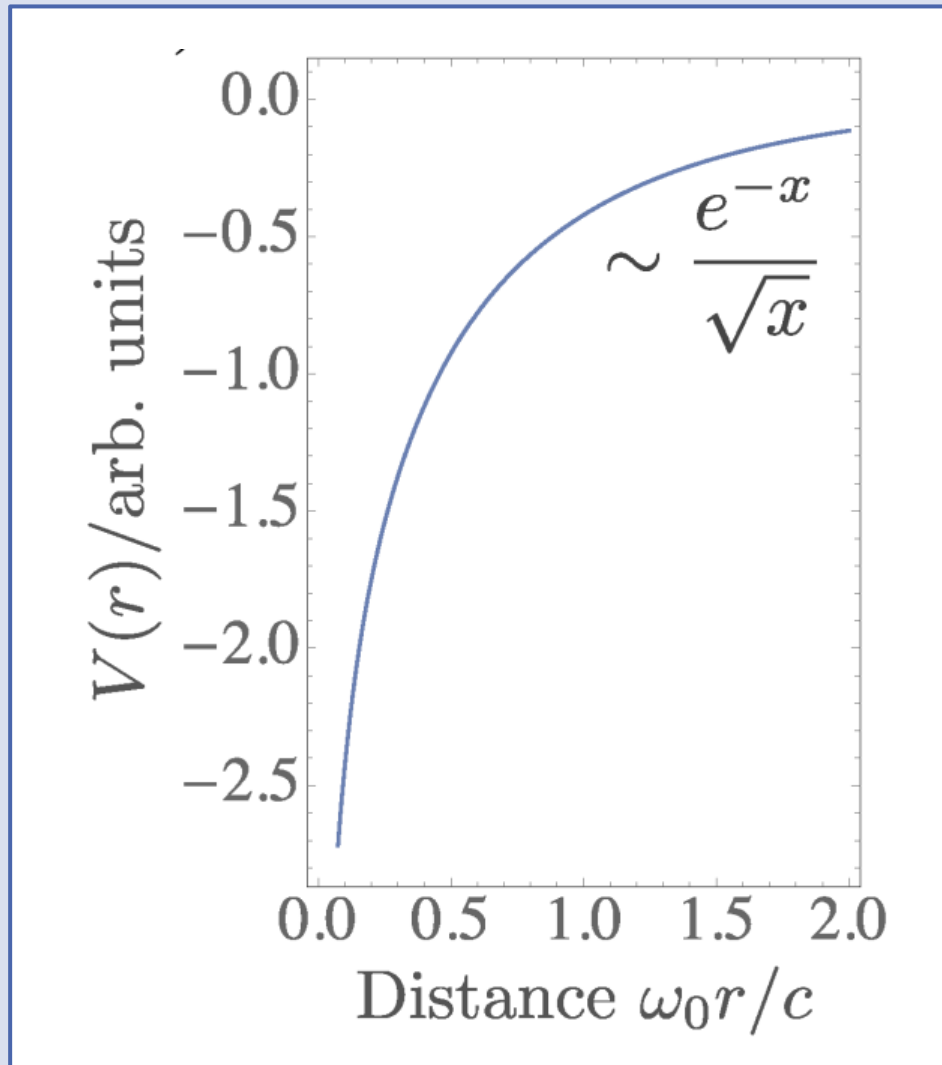
- The overall interaction strength is

$$V_0 = 2g_0^2 \left(\frac{a\omega_0}{c}\right)^2 \frac{1}{\hbar\omega_0}$$

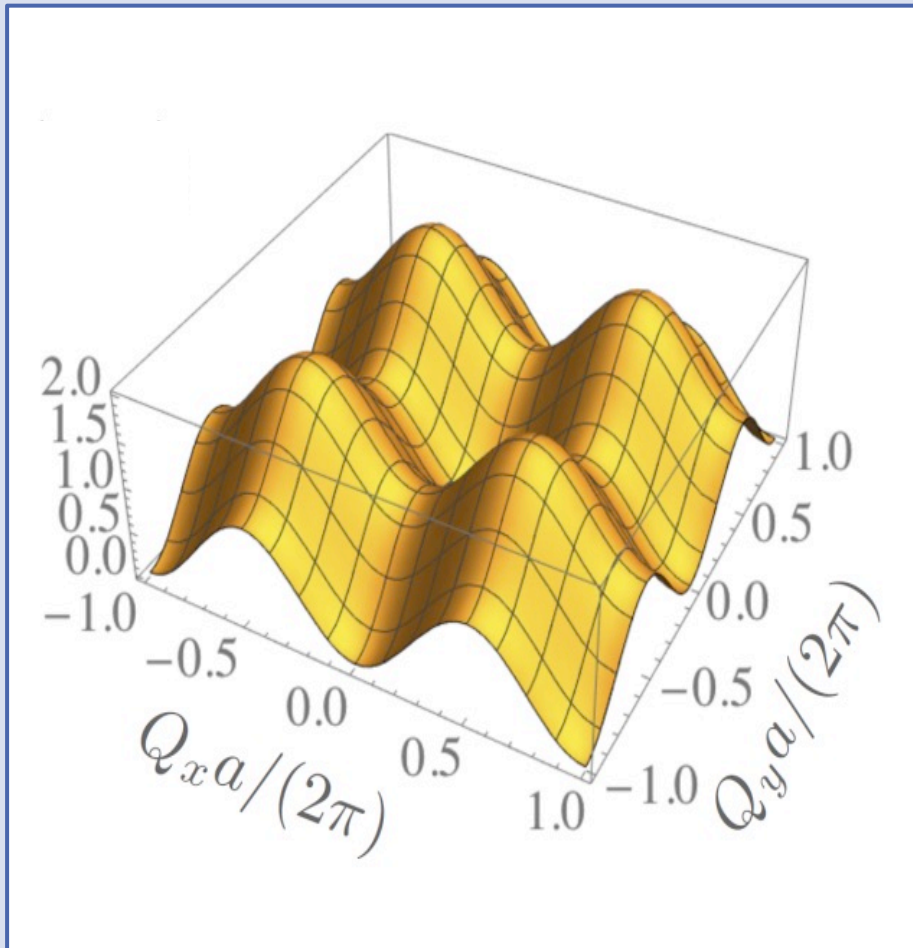
- The bare coupling to the cavity is given by

$$g_0 = t \sqrt{\frac{4\alpha}{\sqrt{\epsilon} A}}$$

- Here A is the cavity compression, α the fine structure constant, a the lattice period, and ϵ the dielectric constant.



- Independent of lattice properties
 - Peaks strongly around $\vec{p} = \vec{p}'$
 - $V(r) \propto -K_0 \left(\frac{\omega_0 r}{c} \right)$
 - Logarithmic divergence in real space (Bessel K_0)
 - Decays over optical wave length scales
- long range interaction



- Only depends on $\vec{p} + \vec{p}'$ and the nesting vector $\vec{Q}$
- Peaks $\vec{Q}_{\max}$ which fulfil $Q_x = \pm Q_y$
- The interactions are attractive only near $\vec{Q}_{\max}$
- We thus decouple e-e interactions and study mean fields around $\vec{Q}_{\max}$

- We define the mean fields around the nesting vectors

$$\Delta_{\sigma\sigma'}^{(\vec{Q})}(\vec{p}) = \frac{1}{N} \sum_{\vec{p}'} V_{\vec{p},\vec{p}'}^{(\vec{Q})} \left\langle c_{\vec{Q}-\vec{p}',\sigma'} c_{\vec{Q}+\vec{p}',\sigma} \right\rangle$$

- We solve the linearized gap equation

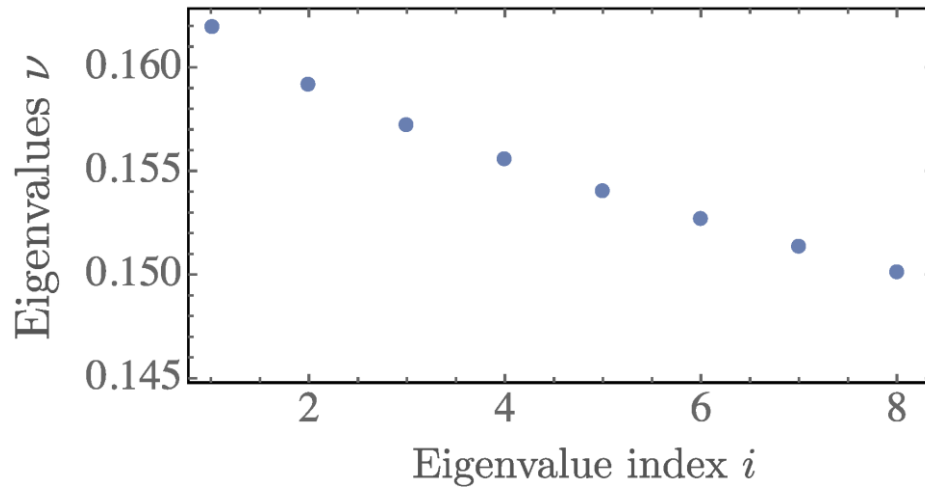
$$-\frac{1}{2} \int_{\vec{Q}+\vec{p}' \in FS} \frac{d\vec{p}'}{(2\pi)^2} \frac{1}{|\vec{v}(\vec{p}')|} V_{\vec{p},\vec{p}'}^{(\vec{Q})} \Delta_{\sigma\sigma'}^{(\vec{Q})}(\vec{p}) = \nu \Delta_{\sigma\sigma'}^{(\vec{Q})}(\vec{p})$$

- to find the largest eigenvalues ν .
- We have also used

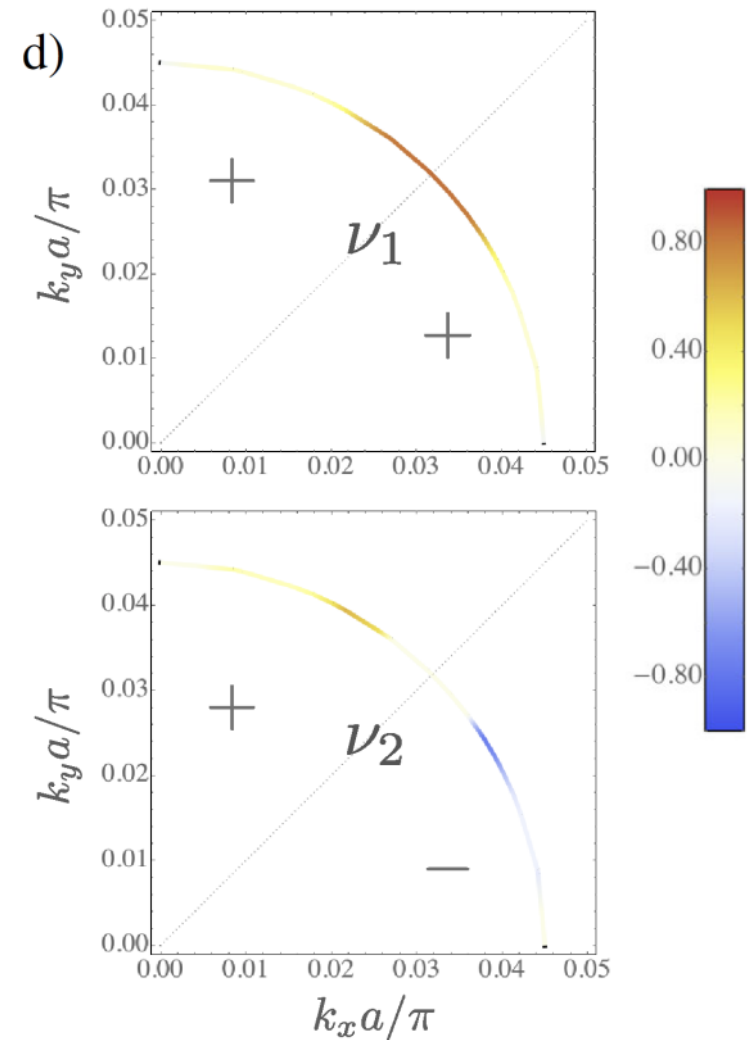
$$2 \vec{v}(\vec{p}') = \nabla_{\vec{Q}+\vec{p}} \epsilon_{\vec{Q}+\vec{p}} + \nabla_{\vec{Q}-\vec{p}} \epsilon_{\vec{Q}-\vec{p}}$$

$$\text{with } \epsilon_{\vec{p}} = -2t [\cos(p_x a) + \cos(p_y a)] - \mu$$

Solutions to the gap equation

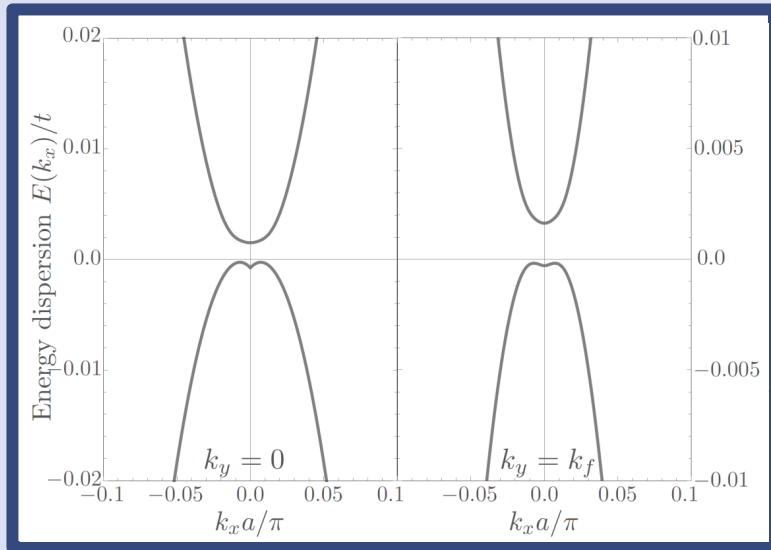


- Eigenvalues translating to $T_c \approx 1K$
- Several nearly degenerate eigenvalues
- Interaction supports singlet and triplet channels

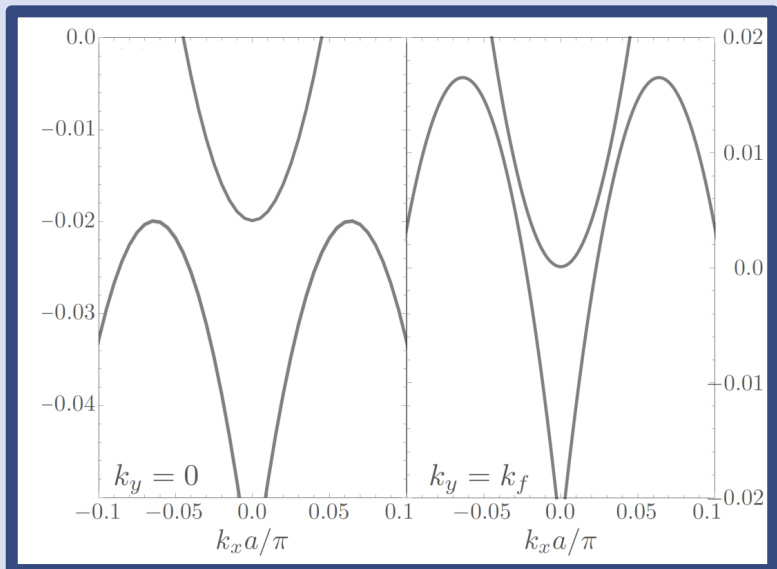


$$\epsilon = 13, m^* = 0,069m_e, \mu = -3.98 t, a = 5.6\text{\AA}, \omega_0 = 2\pi \times 5\text{THz}$$

Quasi particle band structure

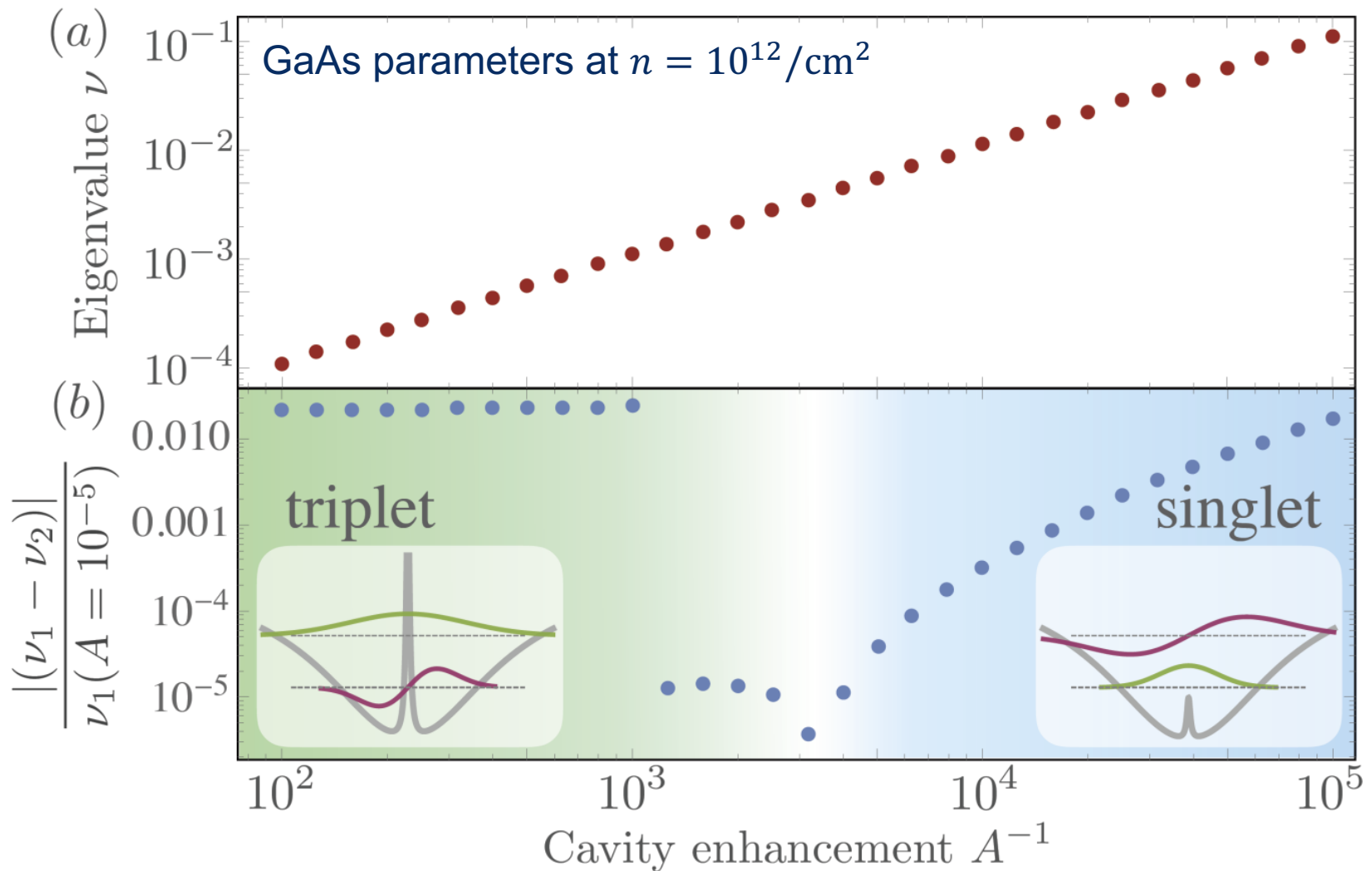


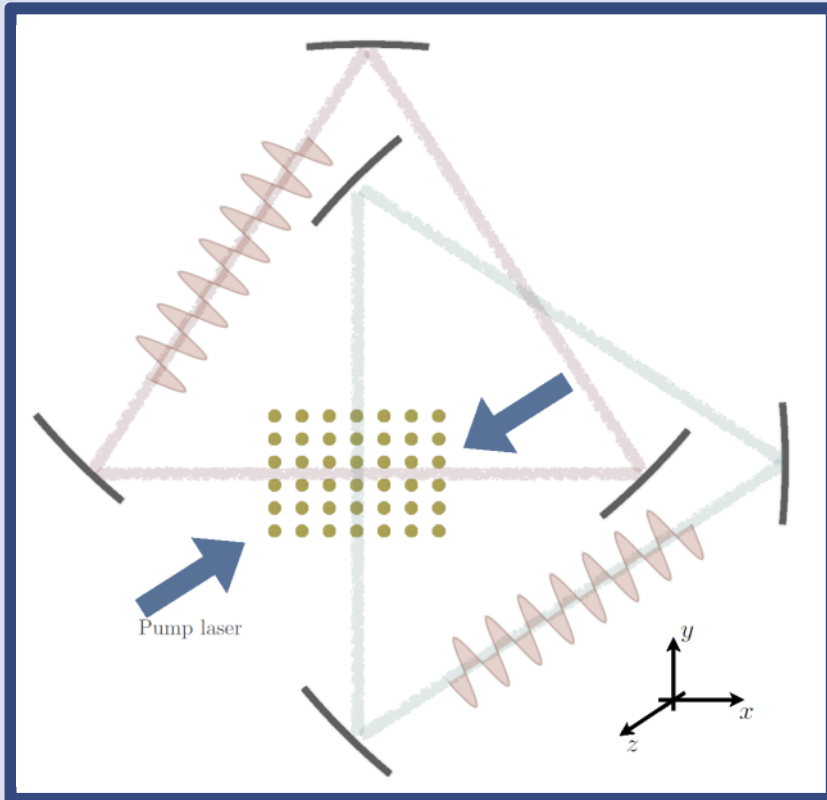
- Small electron density
 - $n = 0.3 \times 10^{11} \text{ cm}^{-2}$
 - Almost isotropic interaction
 - Band gap everywhere



- Larger electron density
 - $n = 2 \times 10^{12} \text{ cm}^{-2}$
 - Anisotropic interaction
 - Band gap closes in regions of small e-e interactions

Including onsite repulsion U





- In the Cooper channel

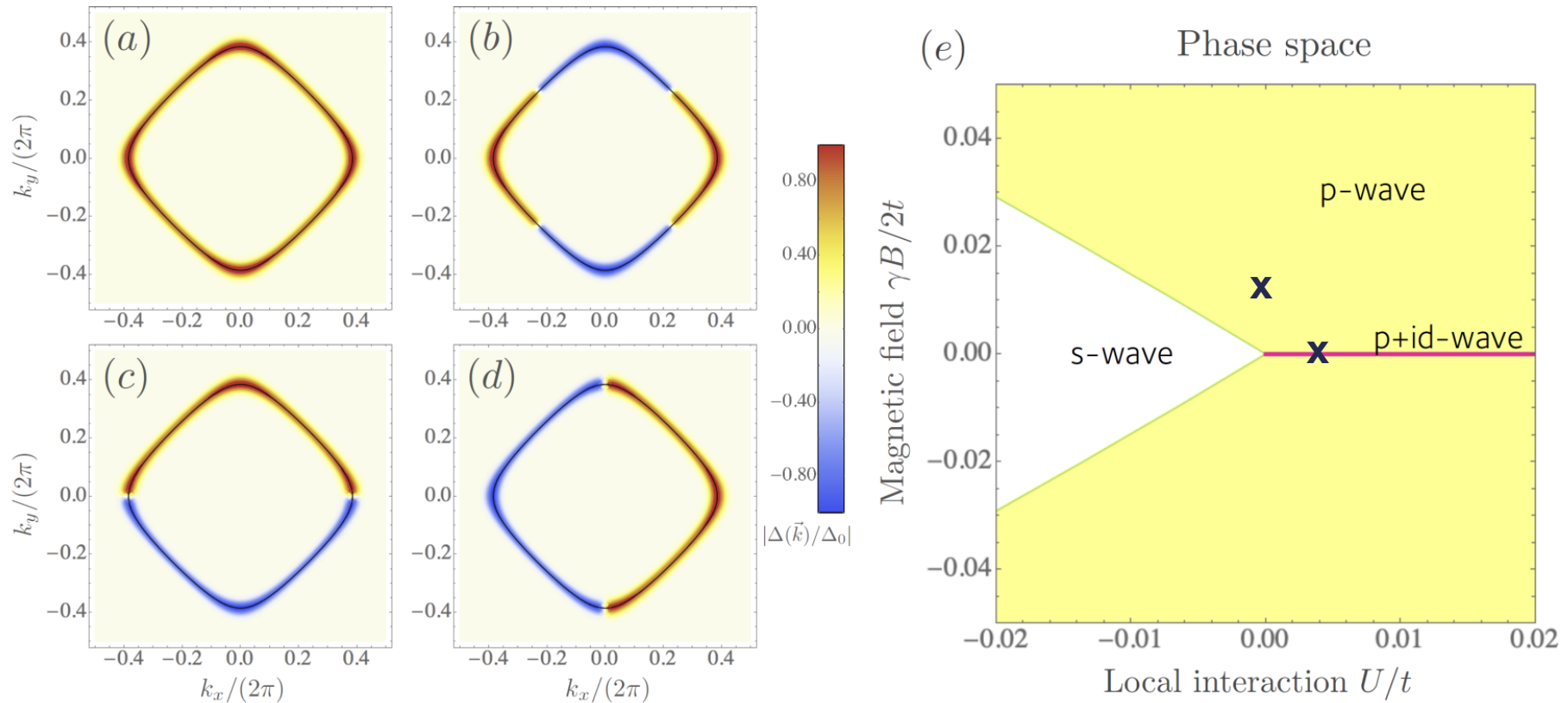
$$V_{\vec{k}\vec{k}'} \propto \sum_{\vec{k}_c} [\delta(\vec{k} - \vec{k}' - \vec{k}_c) + \delta(\vec{k} - \vec{k}' + \vec{k}_c)]$$

- Here k_c is the momentum kick experienced by an atom through scattering a photon laser $\rightarrow$ cavity $\rightarrow$ laser.
- This pairing interaction gives a simple gap equation that has multiple nearly degenerate solutions.

We introduce perturbations U and B to break these degeneracies

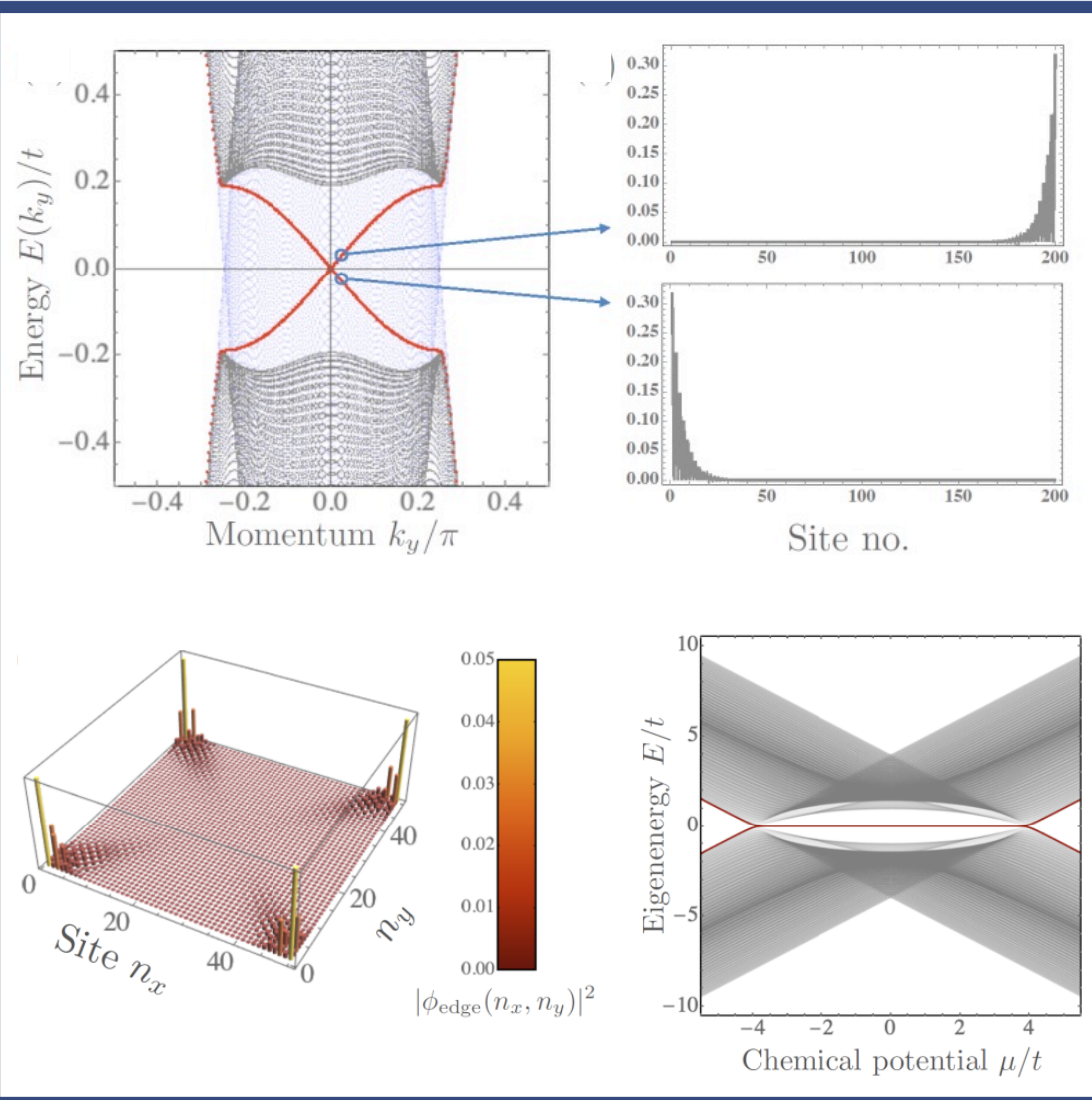
$$H_{\text{Hub}} = -t \sum_{\langle i,j \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{B}{2} \sum_i (n_{i\uparrow} - n_{i\downarrow})$$

Solutions to the gap equation



The shape of the nearly degenerate solutions is determined by lattice geometry

Cavity-induced topological phases



- p -phase with topologically protected edge states
- strip 200 sites, periodic
- minority spin in blue – no gap
- parameters: $B = 0.02t$, $V_0 = -0.1t$, $U = 0$.
- $p + id$ -phase, helical superfluid
- second-order topological state
- localized Majorana states
- parameters: $B = 0$, $\Delta_0 = 0.2t$, $U = 0.02t$

- The diamagnetic interaction is given by

$$H_{\text{dia}} = \frac{e^2}{2m} \sum_{\vec{q}, \vec{q}'} \rho_{\vec{q}} \vec{A}(\vec{q}') \cdot \vec{A}(\vec{q} - \vec{q}')$$

where $\rho_{\vec{q}} = \sum_{\vec{k}, \sigma} c_{\vec{k}+\vec{q}, \sigma}^\dagger c_{\vec{k}, \sigma}$.

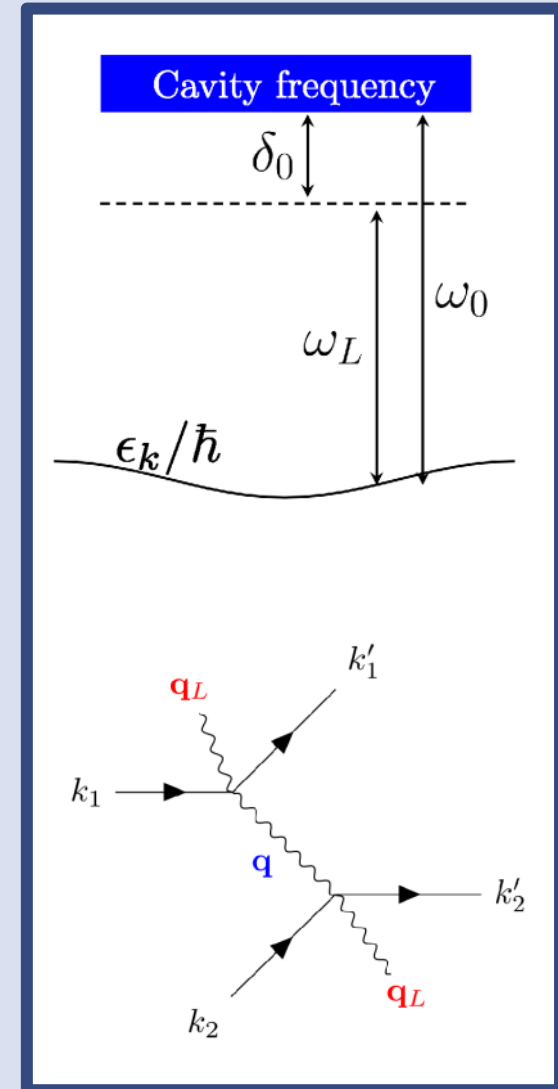
- For an empty cavity this is usually negligible

$$|\gamma_{\vec{k}\vec{q}\vec{q}'}| \sim \frac{t}{A} \frac{4\alpha}{\sqrt{\epsilon}} \left(\frac{a\omega_0}{c} \right)^2 \log \left[1 + \left(\frac{c}{\omega_0} q_{\text{max}} \right) \right] \approx \frac{10^{-8}t}{A}$$

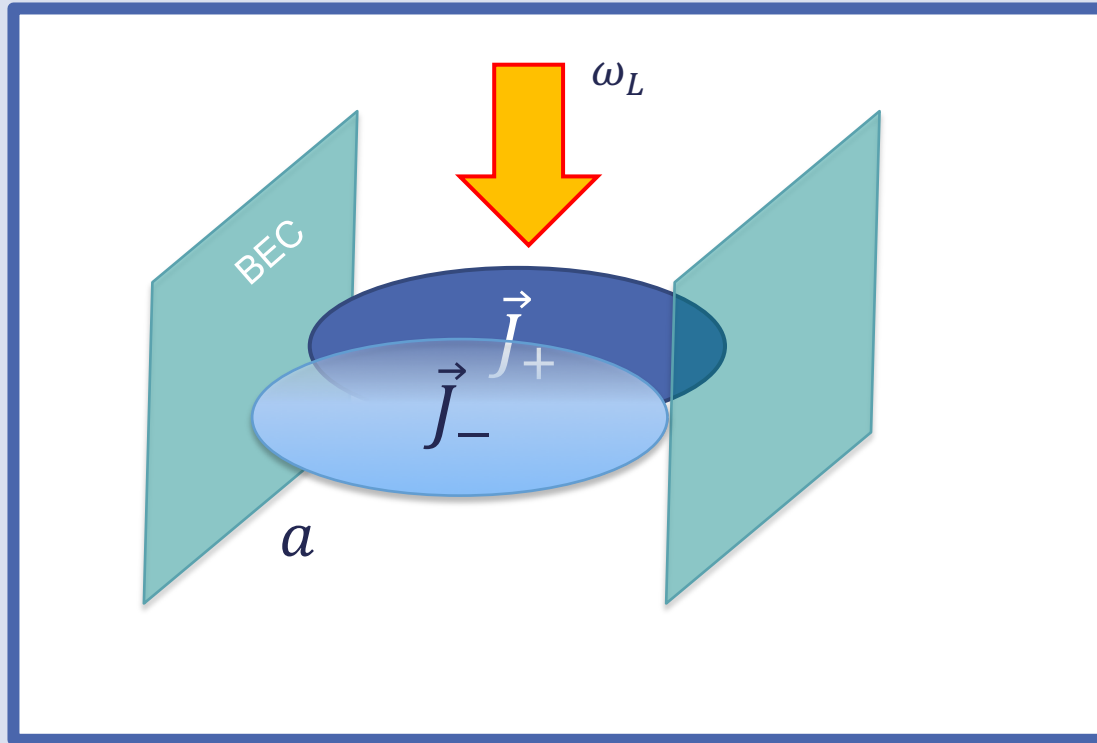
- Driven cavity with $\vec{A} = \vec{A}_{\text{drive}} + \vec{A}_{\text{cavity}}$
- Keep cross-terms and eliminate cavity field

$$H_{\text{eff}} = \sum_{\vec{q}} V(\vec{q}) \rho_{\vec{q}} \rho_{-\vec{q}}$$

$V(\vec{q}) \propto -I \delta_{\vec{q}} (\delta_{\vec{q}}^2 + \kappa_{\vec{q}}^2)^{-1}$ with intensity I , detuning $\delta_{\vec{q}}$.



A two-component BEC in a driven cavity



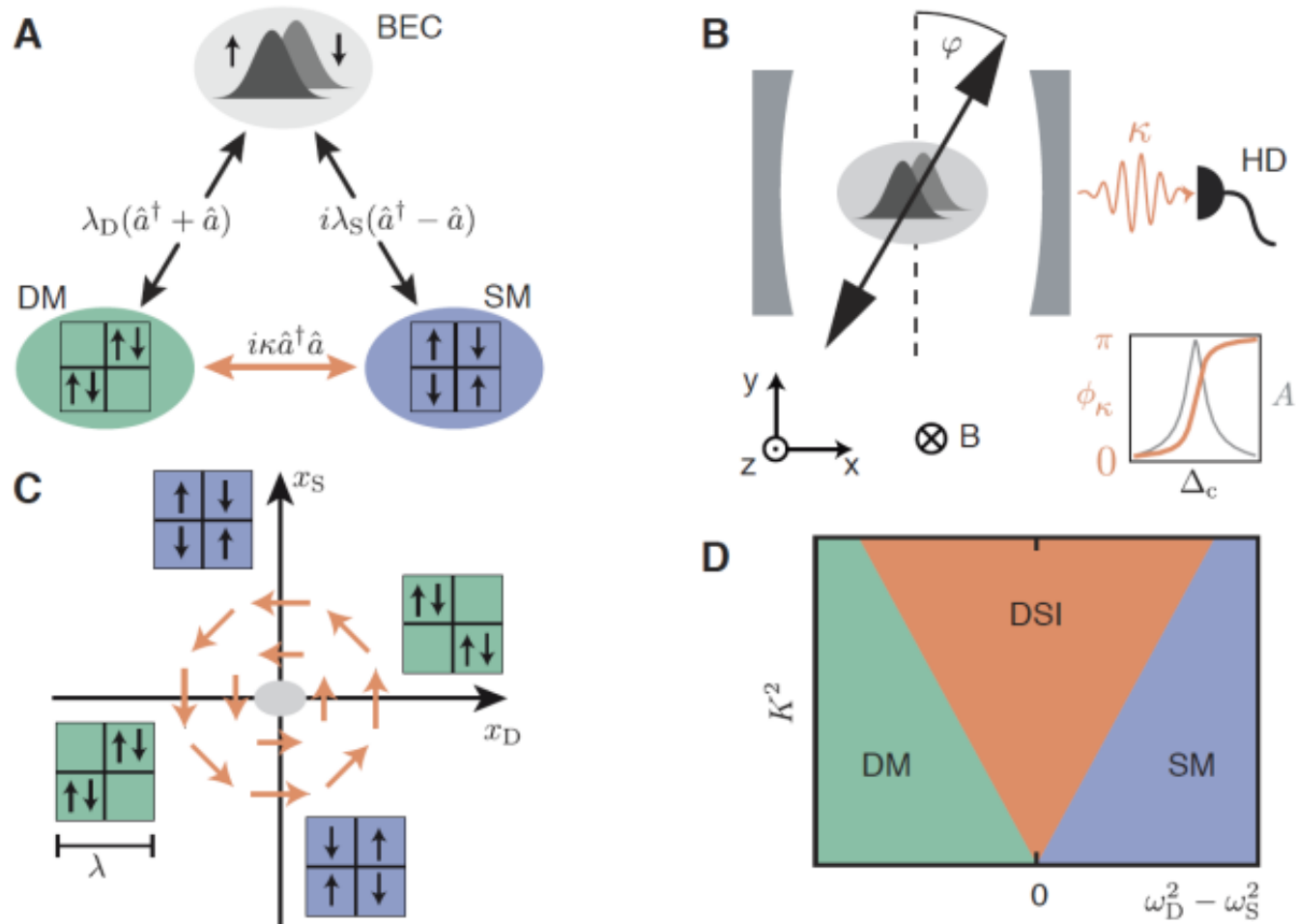
Coupling via atomic
Raman transition

Driving shifts energy scale

Fast cavity dissipation κ

Spin and density mode
couple to different cavity
quadratures

Dissipatively induced instability



Mean field approximation

- Mean field + adiabatic elimination of cavity
- One point functions

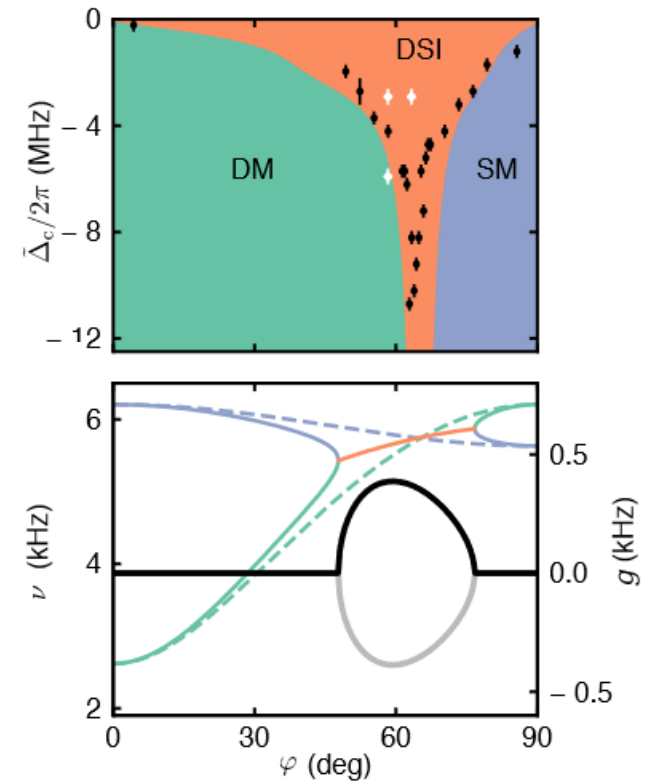
$$x_D = \frac{1}{N} \langle S_{x,+} + S_{x,-} \rangle$$

$$\propto \cos(\omega t) e^{g t}$$

$$x_S = \frac{1}{N} \langle S_{x,+} - S_{x,-} \rangle$$

$$\propto \sin\left(\omega t + \frac{\Delta_c}{\kappa \delta \phi}\right) e^{g t}$$

$$\langle a^{\dagger} \rangle = [A_+ e^{i\omega t} + A_- e^{-i\omega t}] e^{g t}$$



$$\lambda_D = V_{TP} \cos \varphi, \lambda_S = V_{TP} \sin \varphi, \\ \Delta_C = -\omega.$$

Mean field approximation

- Mean field + adiabatic elimination of cavity
- One point functions

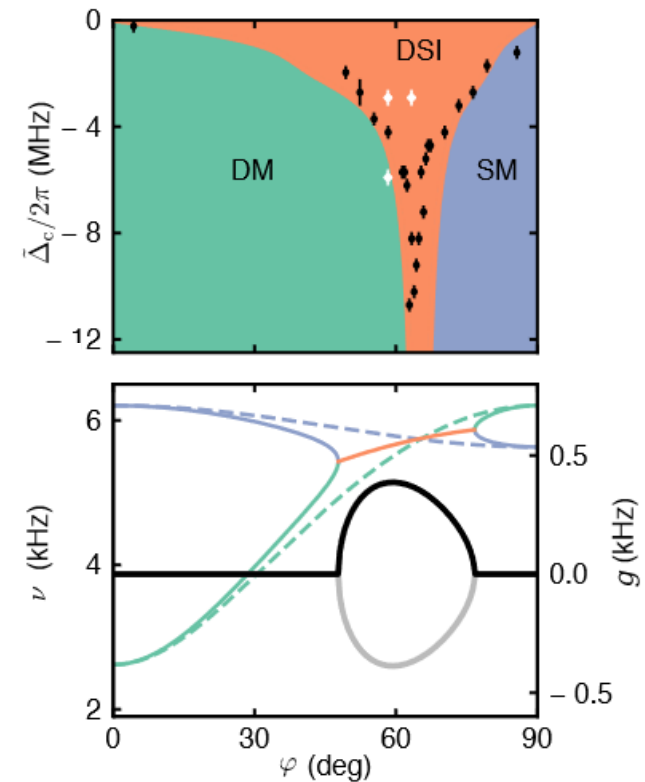
$$x_D = \frac{1}{N} \langle S_{x,+} + S_{x,-} \rangle$$

Some phases oscillate!

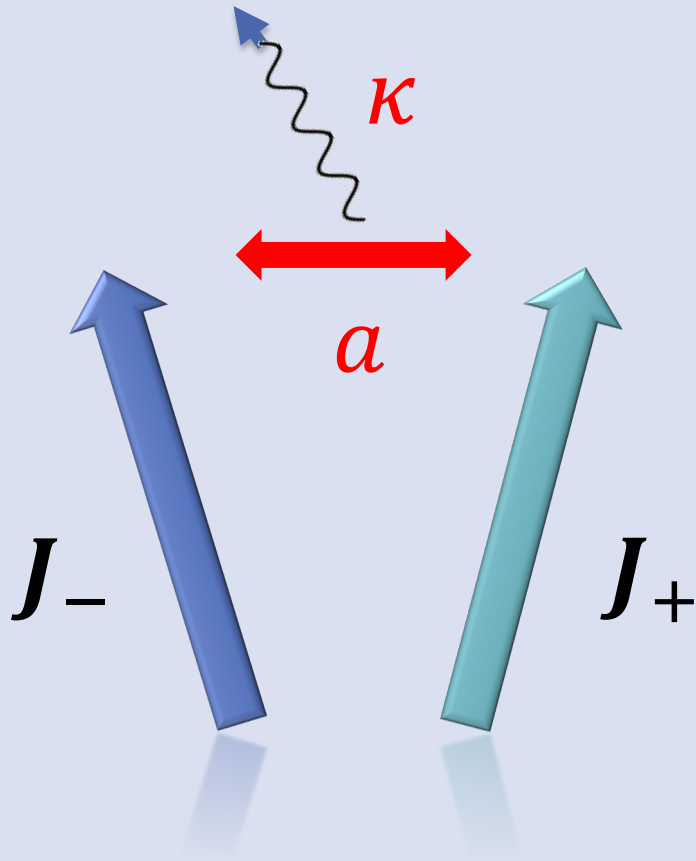
$$\propto \sin\left(\omega t + \frac{\Delta_c}{\kappa \delta \phi}\right) e^{g t}$$

$$I_{\sim} \backslash \text{dag} \backslash = \Gamma A \sim i \omega t + A \sim -i \omega t \sim a t$$

Instability $g > 0$!



$$\lambda_D = V_{TP} \cos \varphi, \lambda_S = V_{TP} \sin \varphi, \\ \Delta_C = -\omega.$$



Hamiltonian

$$H = \hbar\omega a^\dagger a + \hbar\omega_0(J_{z,+} + J_{z,-}) + \frac{\hbar}{\sqrt{N}}[\lambda_D(a^\dagger + a)(J_{x,+} + J_{x,-}) + i\lambda_S(a^\dagger - a)(J_{x,+} - J_{x,-})]$$

Dissipation $L = a$

$$\dot{\rho}(t) = \mathcal{L}\rho(t) = -i[H, \rho(t)] + \kappa(2a\rho(t)a^\dagger - \{a^\dagger a, \rho(t)\})$$

- Large number of atoms $N \gg 1$

$$S_{+,\pm} = b_{1,2}^\dagger \sqrt{N - b_{1,2}^\dagger b_{1,2}}$$

$$S_{z,\pm} = b_{1,2}^\dagger b_{1,2} - \frac{N}{2}$$

- Expand in N (normal phase)

$$H = \hbar\omega a^\dagger a + \hbar\omega_0(b_1^\dagger b_1 + b_2^\dagger b_2) + \hbar\lambda_D(a + a^\dagger)(b_1 + b_1^\dagger + b_2 + b_2^\dagger) + i\hbar\lambda_S(a - a^\dagger)(b_1 + b_1^\dagger - b_2 - b_2^\dagger)$$

- Include shift for superradiant phase

$$a \rightarrow a + \alpha\sqrt{N}$$

$$b_{1,2} \rightarrow b_{1,2} - \sqrt{\beta_{1,2}N}$$

- Quadratic equation in bosonic operators

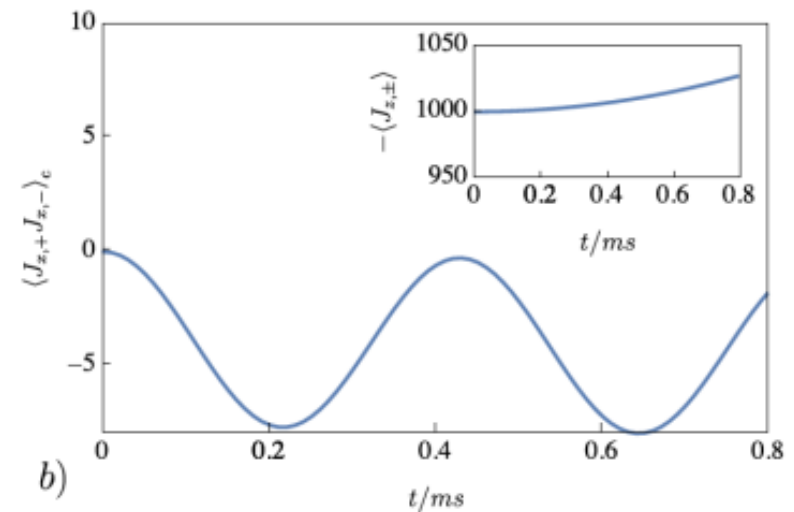
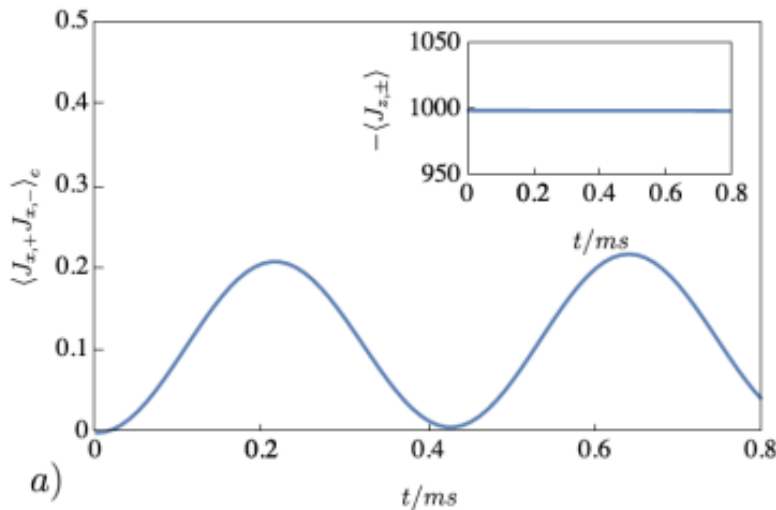
Exact solution in the thermodynamic limit

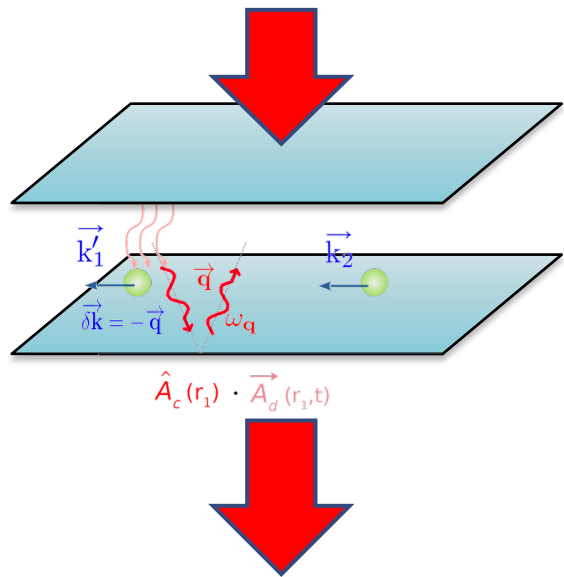


- We solve the resulting system exactly using third quantization.
- We find for the eigenvalues λ_{n_1, n_2} with $n_{1,2} = 0, \pm 1, \pm 2, \dots$

$$\lambda_{n_1, n_2} = i n_1 \omega_0 + \frac{2n_2}{\kappa} (\lambda_D^2 + \lambda_S^2) + O\left(\frac{1}{\kappa^2}\right)$$

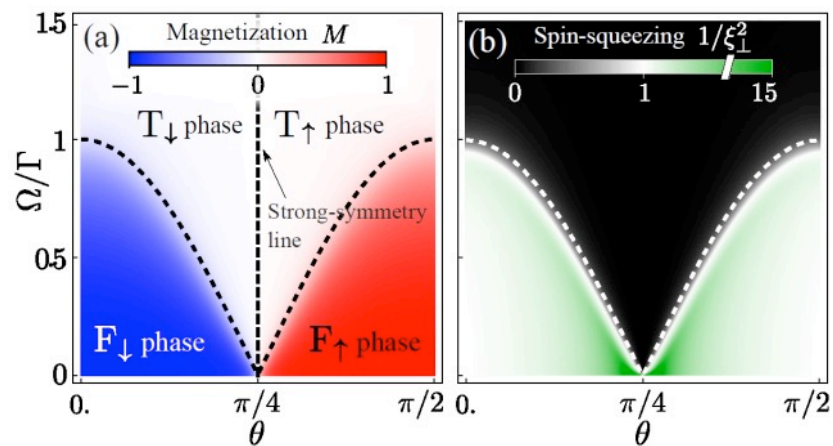
- This shows that there is always an unstable mode (not necessarily mean fields).





A^2 terms in driven cavities

- Driving-induced interactions
- Non-classical optical fields
- Strong coupling regime



Spontaneous trajectory freezing

- No intermittency
- Quantum enhanced sensing

Thanks to the team and collaborators



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