

An exceptional point as a dynamical critical point between driven polariton condensates: polaritons as “active matter”

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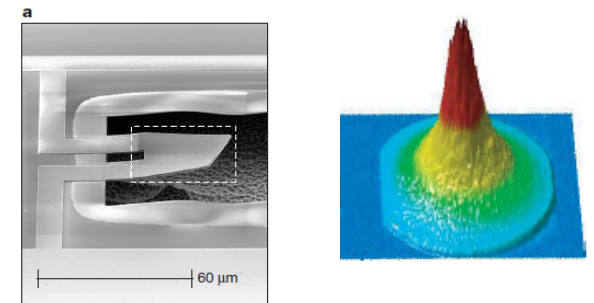
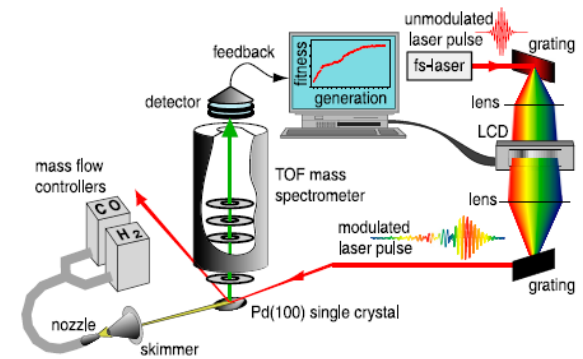
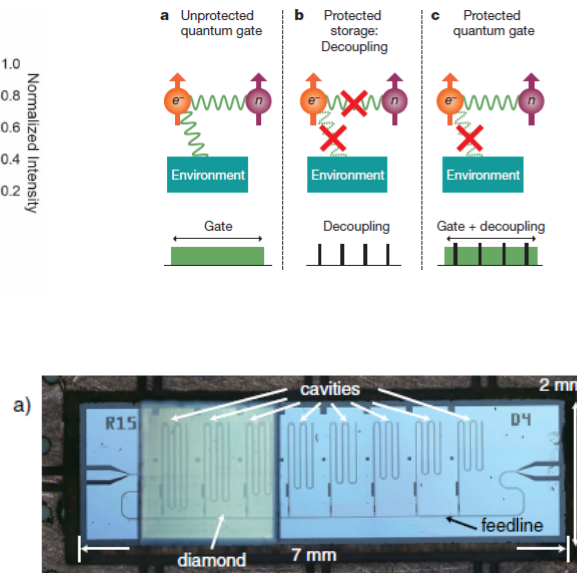
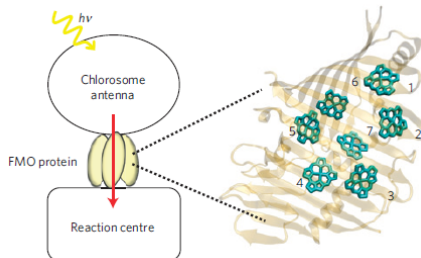
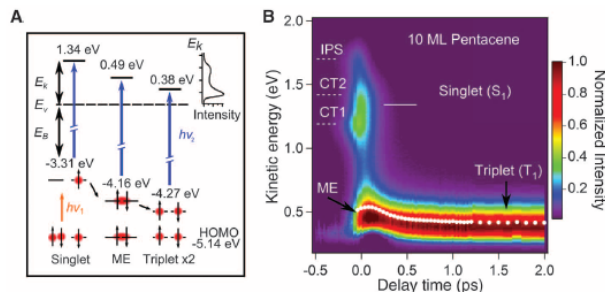
Michel Fruchart (Chicago); Vincenzo Vitelli (Chicago)

Richard Brierley (Nature); Cele Creatore (Cambridge); Marzena Szymanska (UCL); Francesca Marchetti (Madrid); Jonathan Keeling (St Andrews); Paul Eastham (TCD); Sahinur Reja (Indiana)



Quantum Engineering: Any more room at the bottom?

- Not just quantum computing and cryptography : quantum control in general
- 21st century engineering discipline
- Impinges on bio, nanoscience, sensors and detectors, quantum matter, atomic physics, chemistry, computing
- Cavity QED systems couple local quantum objects at long range with resonant and tunable interactions – inherent flexibility





Christiaan Huygens 1629-95

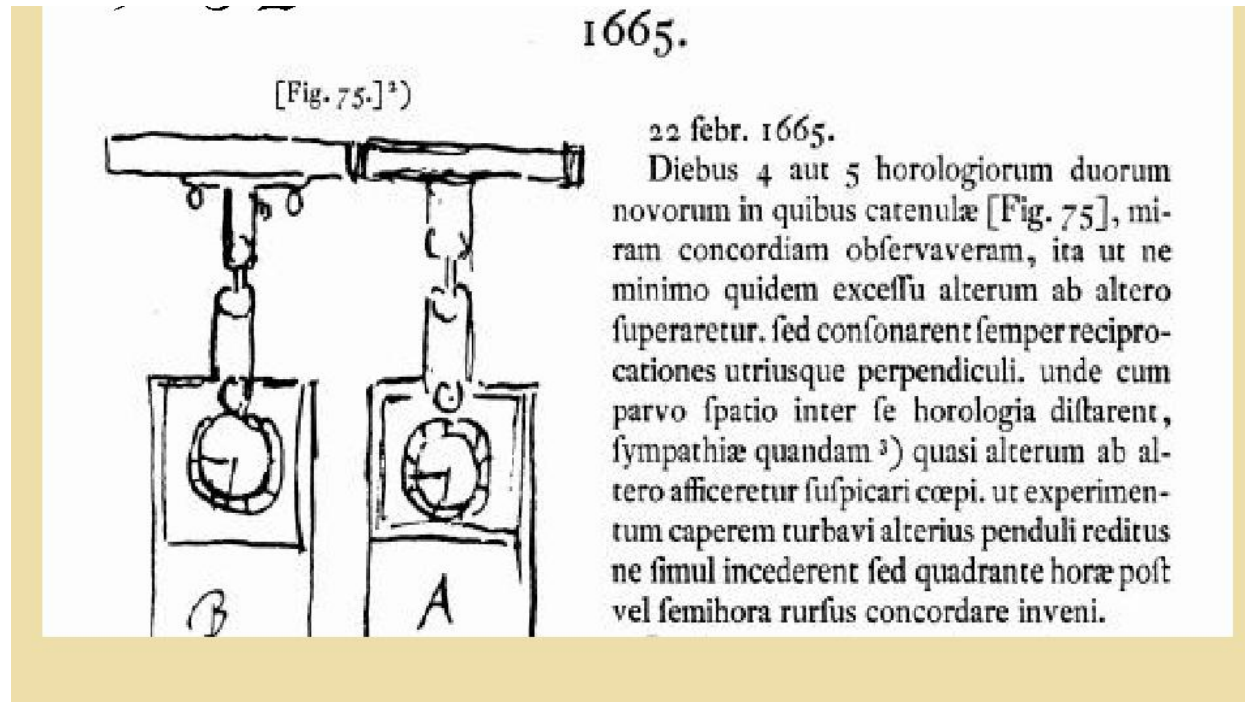
1656 – Patented the pendulum clock

1663 – Elected to Royal Society

1662-5 With Alexander Bruce, and sponsored by the Royal Society, constructed maritime pendulum clocks – periodically communicating by letter

Huygens Clocks

In early 1665, Huygens discovered "...an odd kind of sympathy perceived by him in these watches [two pendulum clocks] suspended by the side of each other."

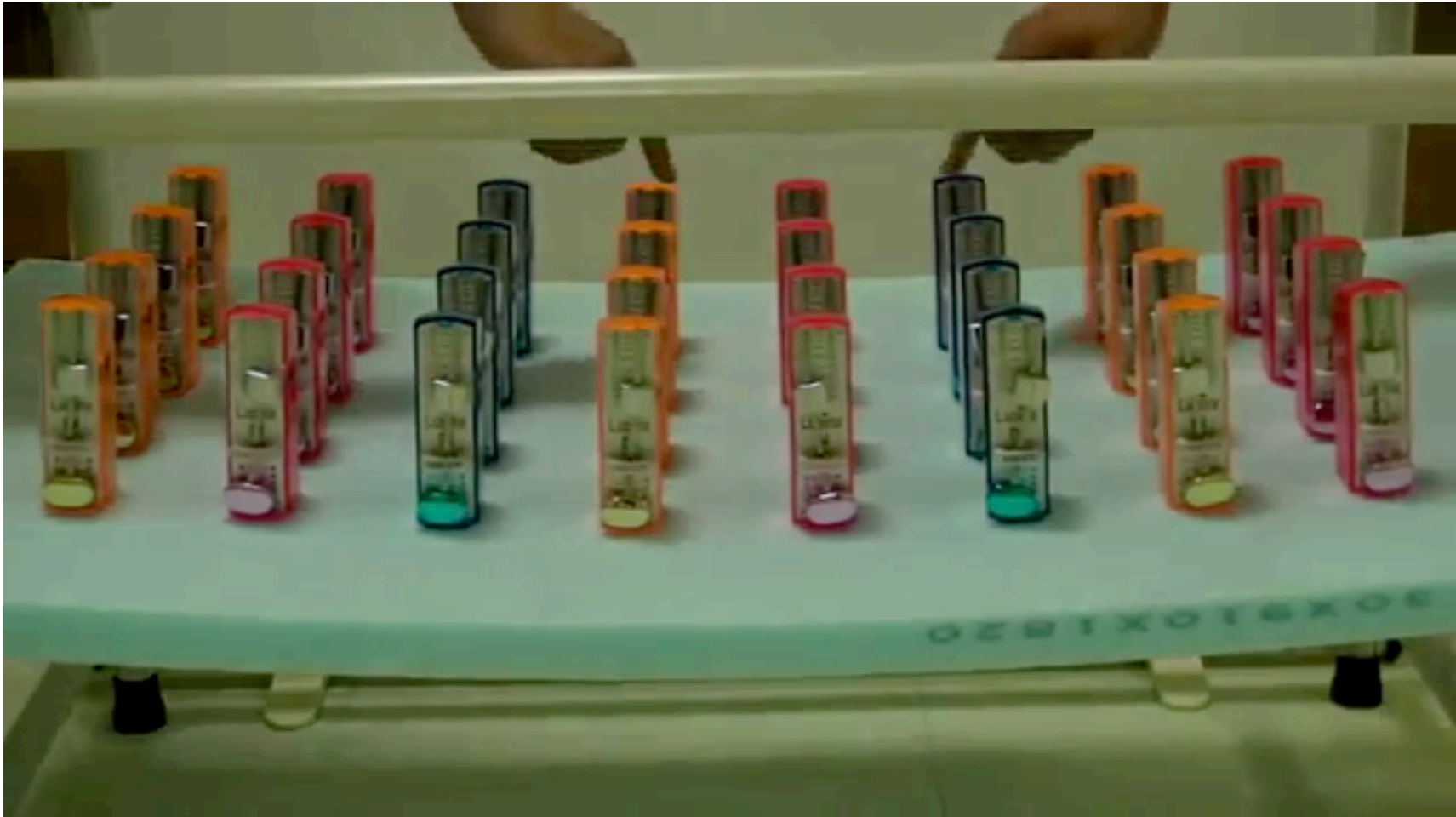


He deduced that effect came from "imperceptible movements" of the common frame supporting the clocks

"Huygens' Clocks", M. Bennett et al, Proc. Roy. Soc. A **458**, 563 (2002)

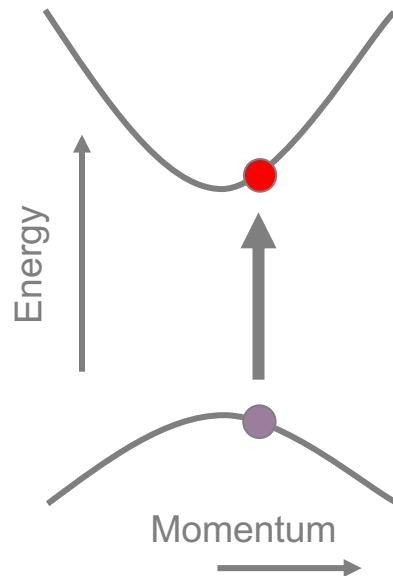
Multiple metronomes

Useful videos at <http://www.youtube.com/user/IkeguchiLab>



Making light atoms inside a solid

Excite electron-hole pair across a semiconductor band gap

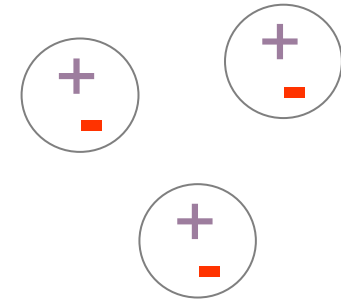


Bound by the screened Coulomb interaction to make an exciton

$$Ry^* = \frac{m^*/m}{\epsilon^2} \times \text{Rydberg}$$

$$a_0^* = \frac{\epsilon}{m^*/m} \times a_{Bohr}$$

Polariton Effective Mass $m^* \sim 10^{-4} m_e$
 $T_{BEC} \sim 1/m^*$

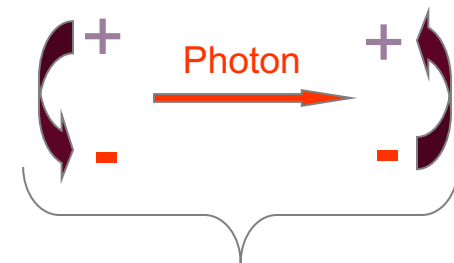


Excitons are the solid state analogue of positronium

In GaAs

Binding energy ~ 5 meV

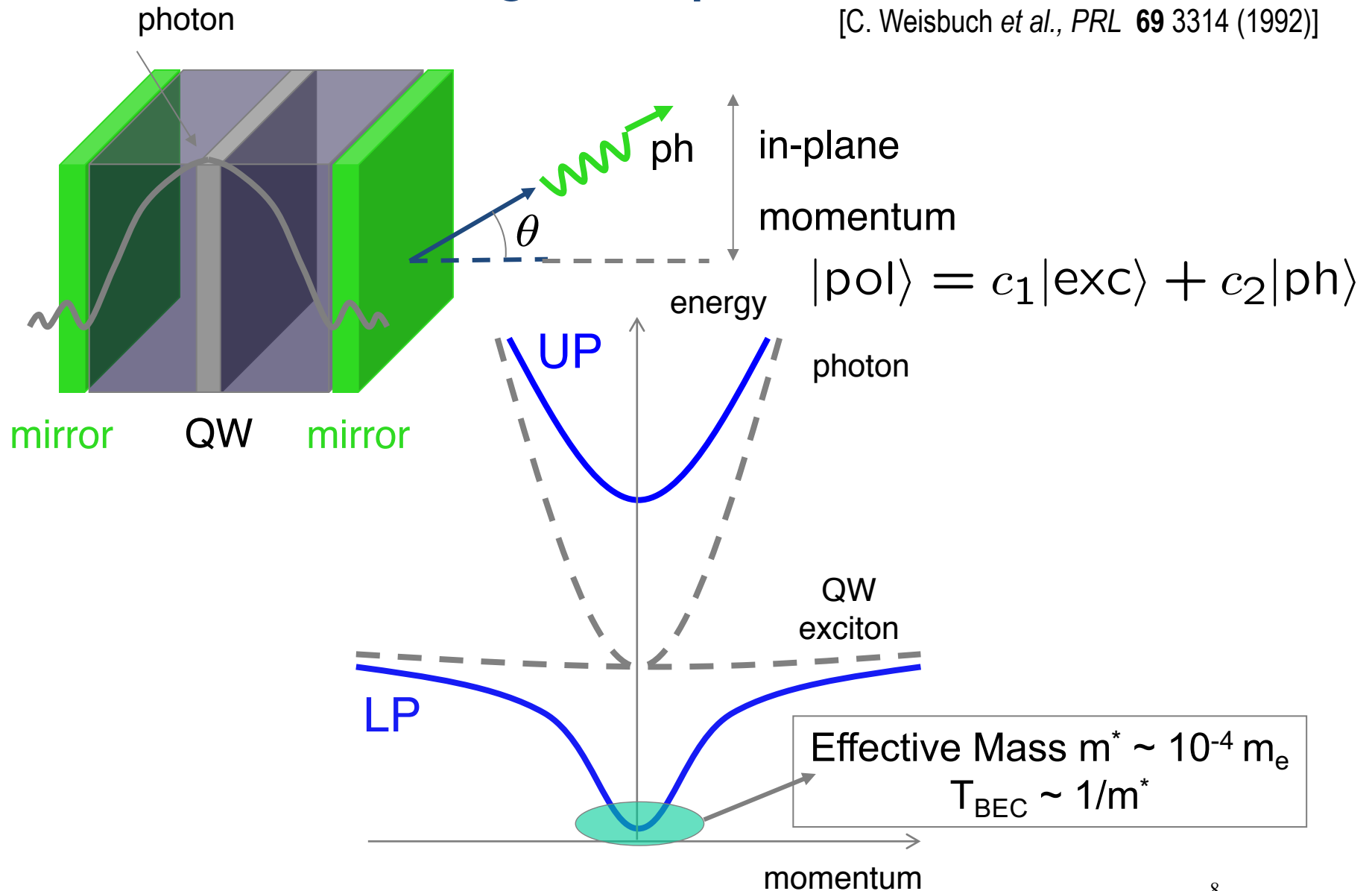
Bohr Radius ~ 7 nm



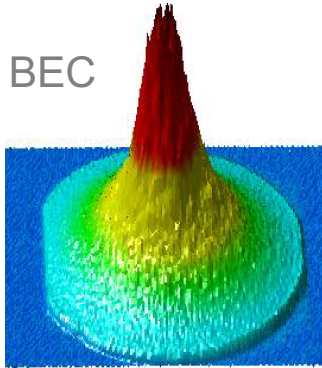
Combined coherent excitation is called a **polariton**

Polaritons: Matter-Light composite bosons

[C. Weisbuch *et al.*, *PRL* **69** 3314 (1992)]

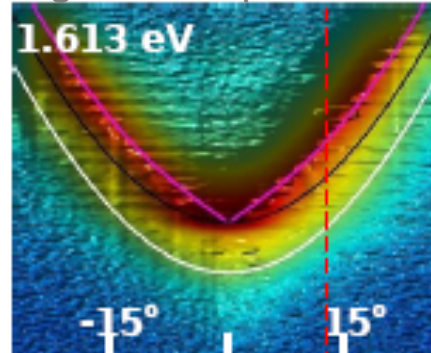


BEC



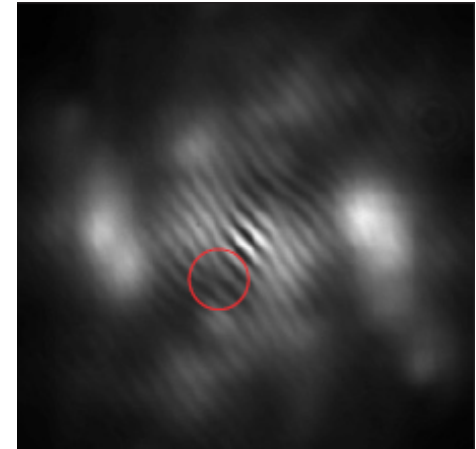
Kasprzak et al 2006

Bogoliubov spectrum



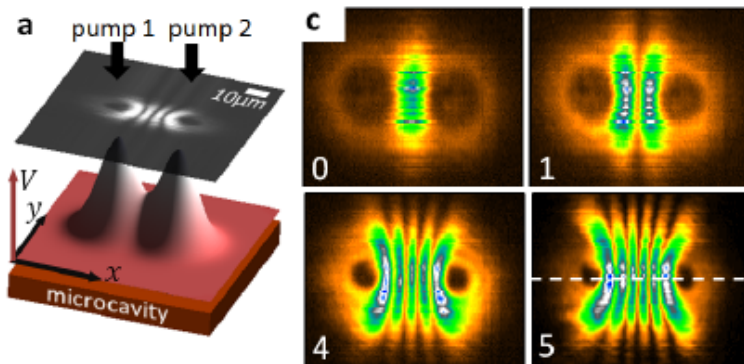
Utsunomiya et al, 2008

Vortices



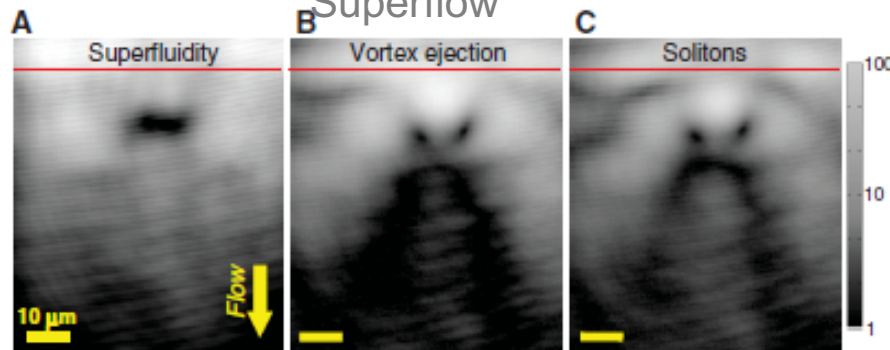
Lagoudakis et al 2008

Coupled condensates



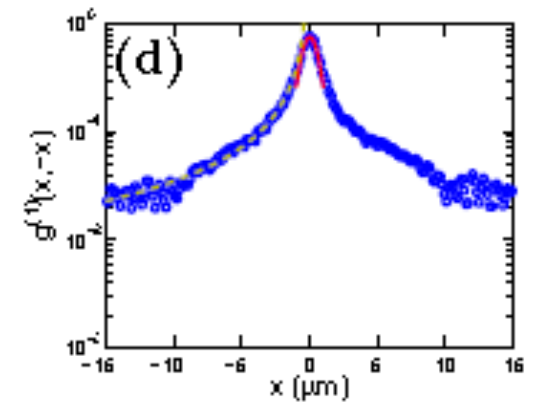
Tosi et al 2012

Superflow



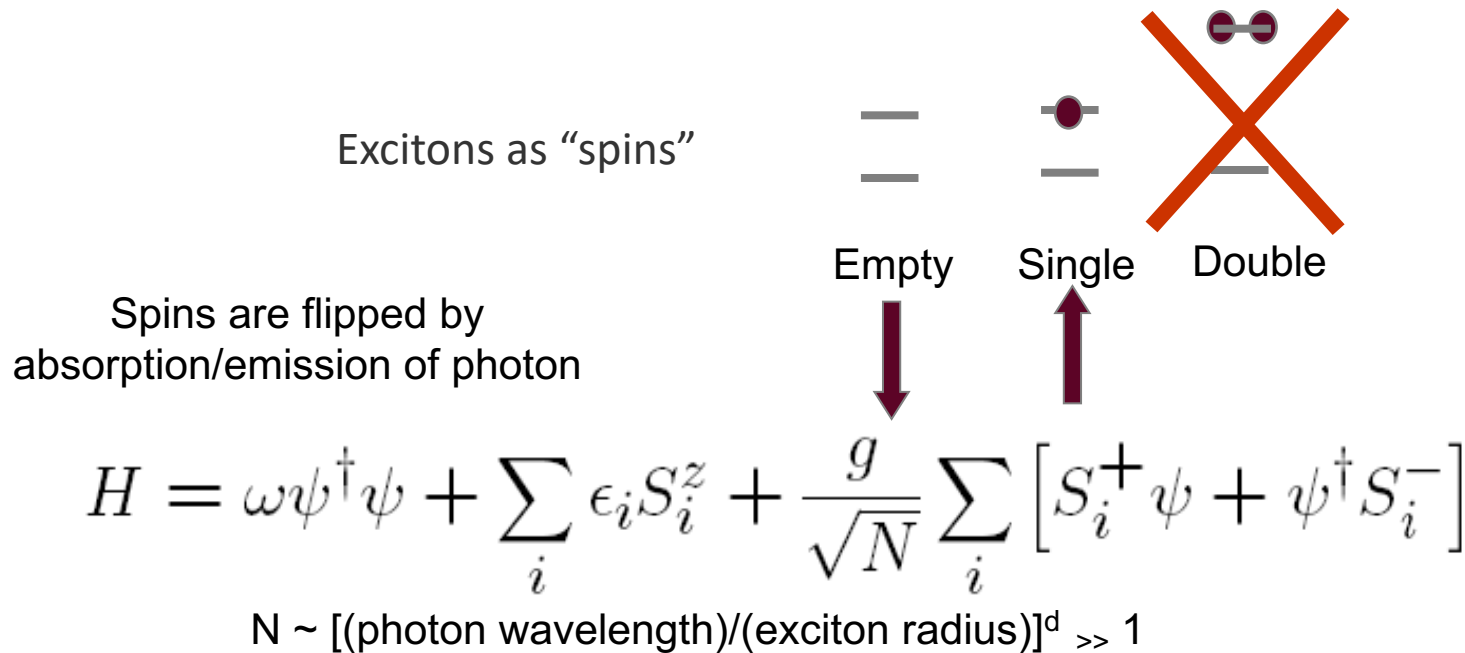
Åmo et al, 2011

Power law correlations



Roumpou et al 2012

Polaritons and the Dicke Model - a.k.a Jaynes-Tavis-Cummings model



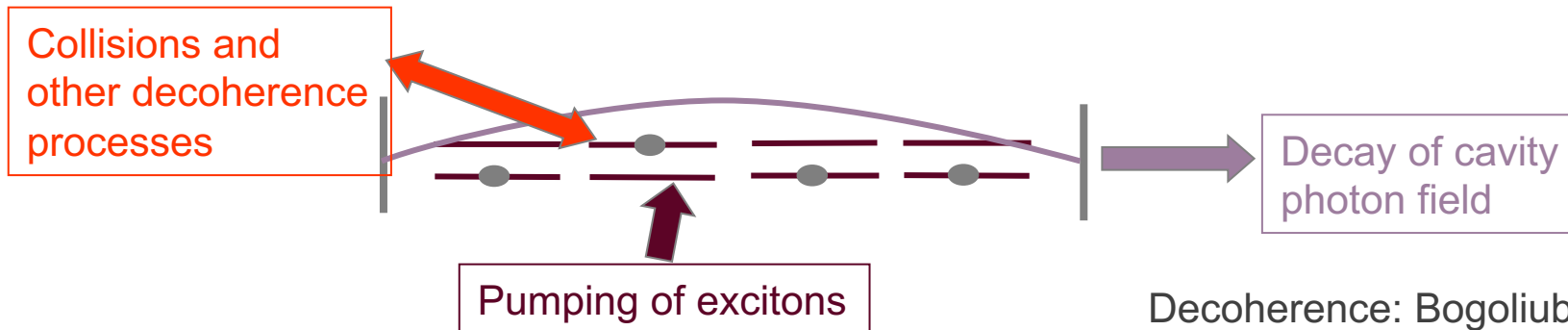
This model is the plain vanilla description of many systems: cavity QED in superconducting systems; coupled quantum dots; impurities in semiconductors; cold Rydberg atoms

Mean field theory – i.e. BCS coherent state – expected to be good approximation

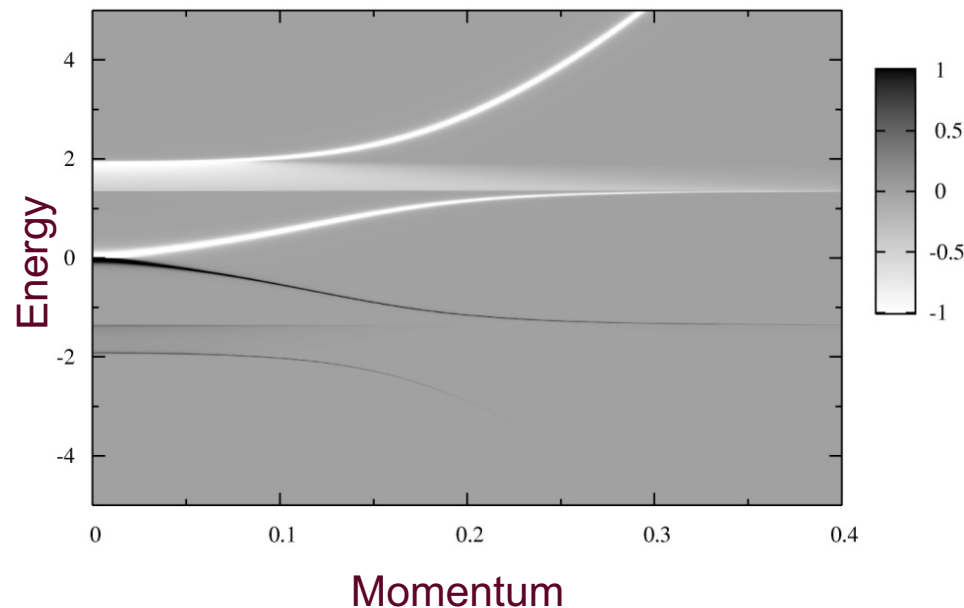
$$|\lambda, w_i\rangle = \exp \left[\lambda \psi^\dagger + \sum_i w_i S_i^+ \right] |0\rangle \quad T_c \approx g \exp(-1/gN(0))$$

Transition temperature depends on coupling constant

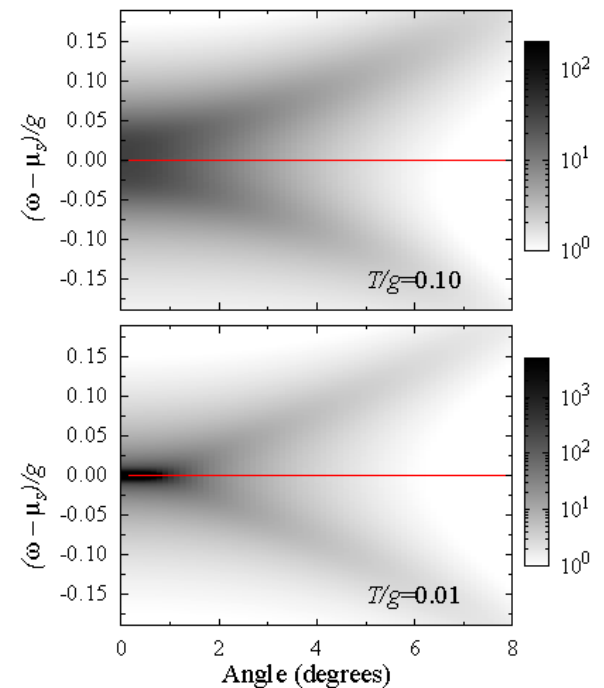
Multiple channels, disorder, and decoherence



Excitations: Bogoliubov mode from lower polariton and “ghost” branches; non-dispersing localized states

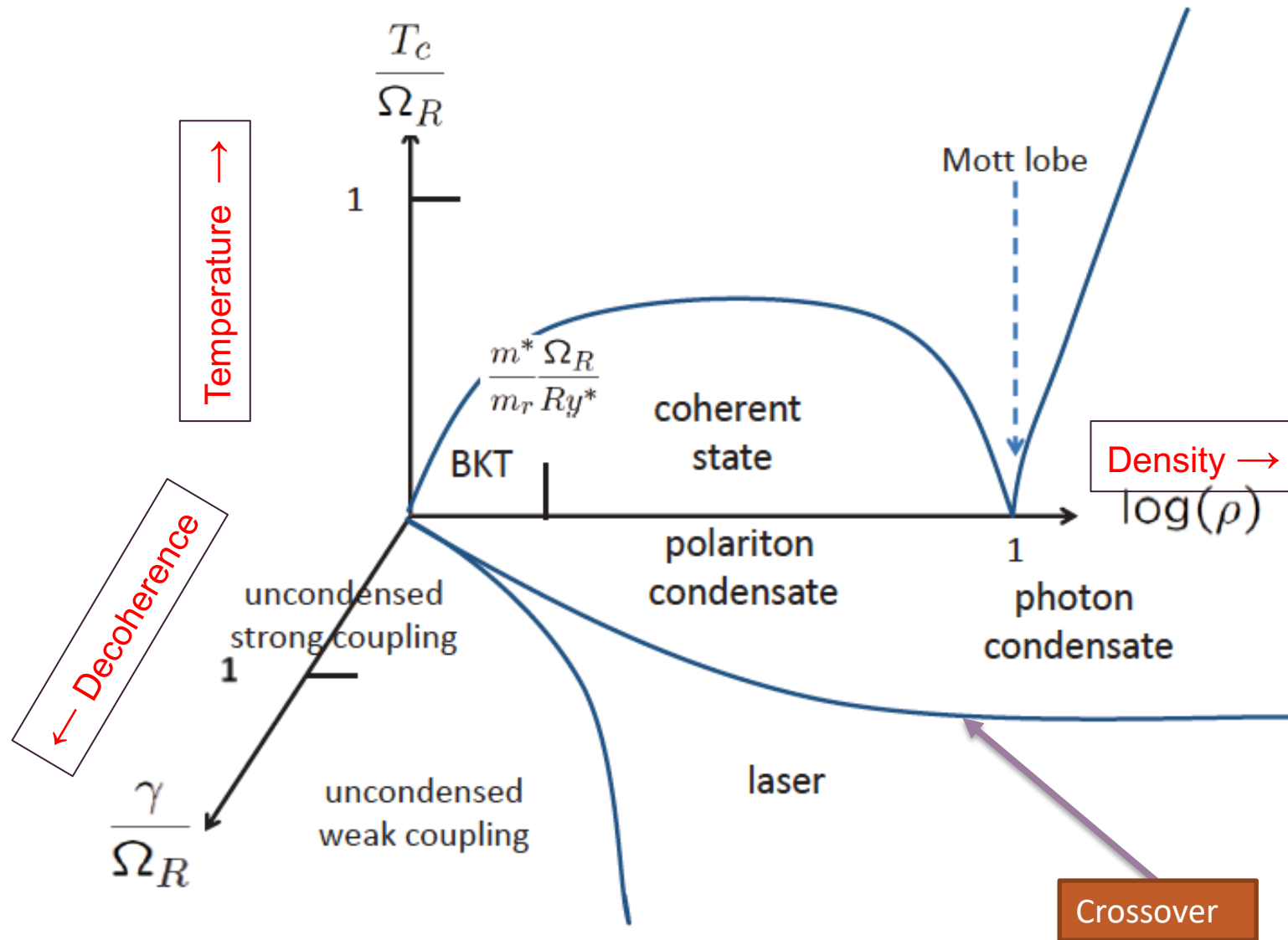


Decoherence: Bogoliubov mode becomes diffusive at small q



Keeling, Eastham, Szymanska, PBL. Phys. Rev. Lett. 93, 226403 (2004)
Szymanska, Keeling, PBL, Phys. Rev. Lett. 96 230602 (2006)

Simplified polariton phase diagram



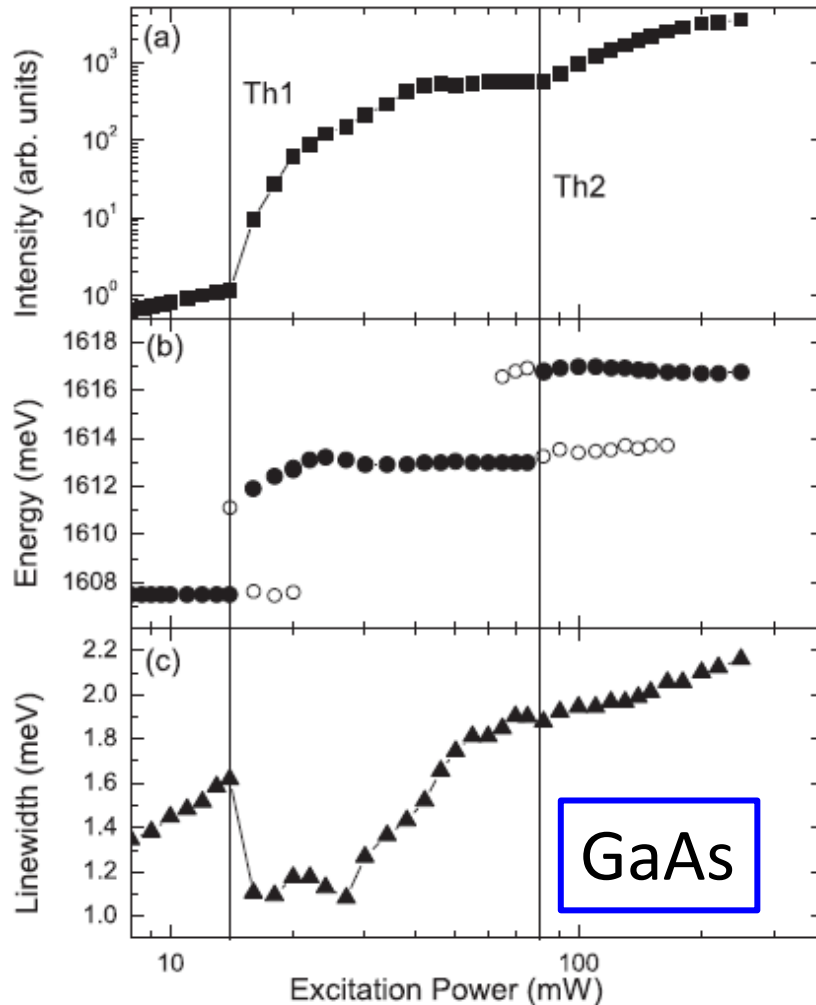
Non-equilibrium condensation

There is no clear distinction between a polariton condensate and a laser

- both have macroscopic population of a photon mode
- in the polariton limit, the excitons are also coherent and there is a single particle gap
- in the laser limit, the coherent exciton population becomes small, and there are gapless single particle excitations
- no phase transition but a crossover between the two ?

One can certainly conceive of building coherent condensates that are either to be thought of as “in phase” (lower polariton) or “out of phase” (upper polariton) Particularly with detuning the LP condensate is excitonic, and the UP condensate is more like a laser

Two-threshold behavior



D. Bajoni, et al., PRL **100**, 047401 (2008).

R. Balili, et al., PRB **79**, 075319 (2009)

P. Tsotsis, et al., New J. Phys. **14**, 023060 (2012).

T. Horikiri, et al., J. Phys. Soc. Jpn. **82**, 084709 (2013).

S. Brodbeck, et al., PRL **117**, 127401 (2016).

S. Kim, et al., PRX **6**, 011026 (2016).

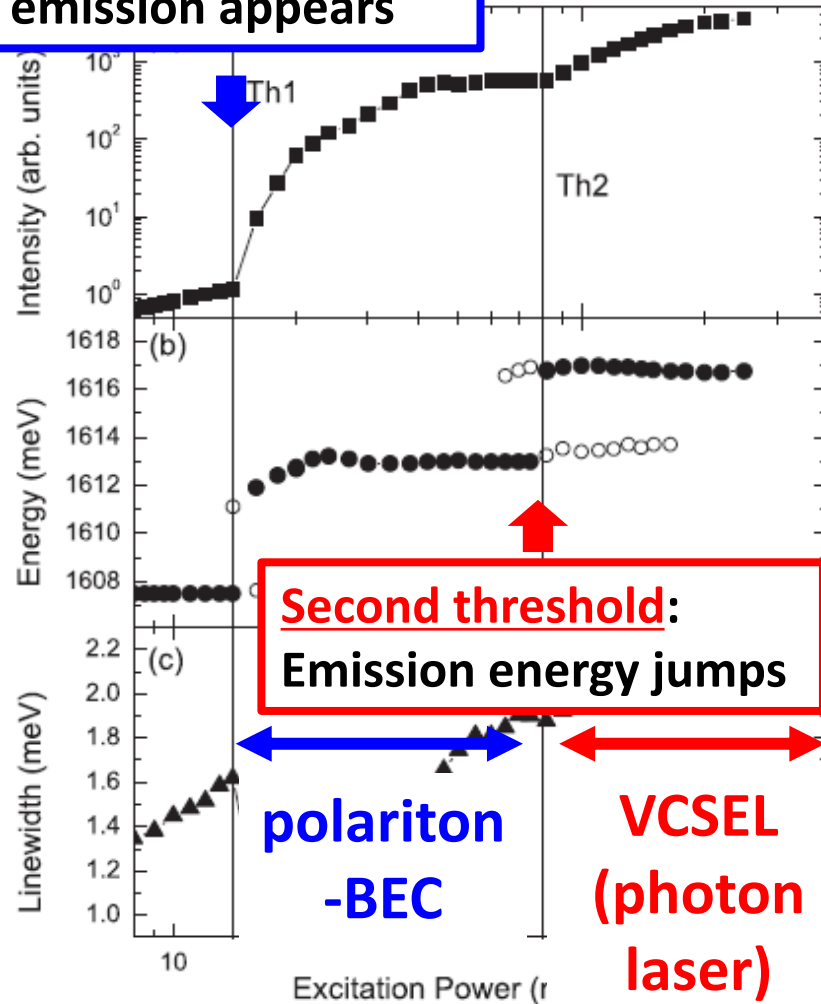
C. P. Dietrich et al., Sci. Adv. **2**, e160666 (2016).

J. Tempel et al., PRB **85**, 075318 (2012).

Two-threshold behavior

First threshold:

Condensate
emission appears



“

Second threshold

= strong-to-weak coupling
transition

”

Normal = $U(1)$ -symmetric

Polariton-BEC = $U(1)$ -broken

Photon laser = $U(1)$ -broken

... no new broken symmetries!

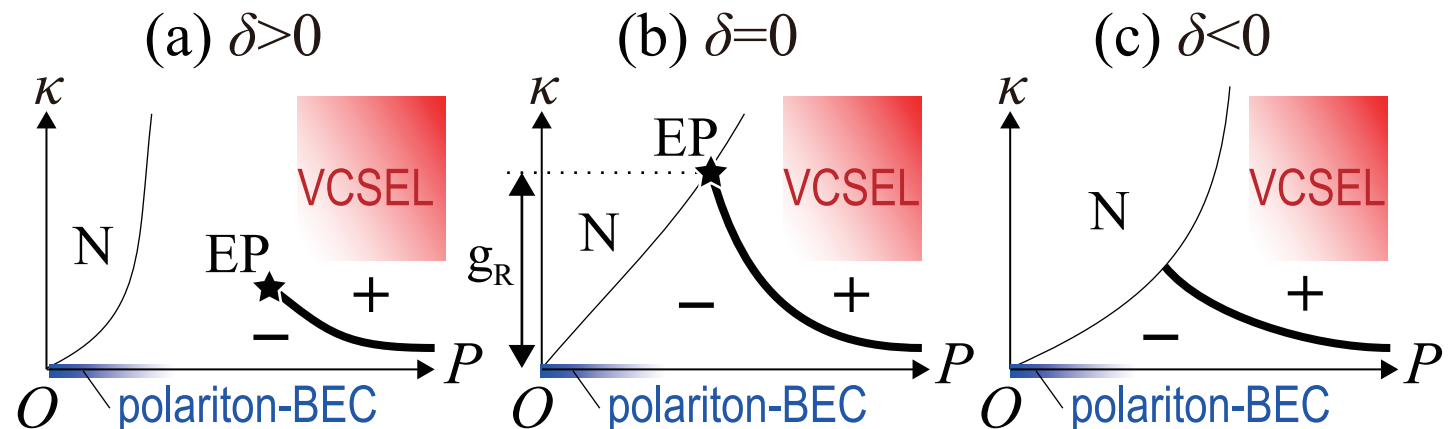
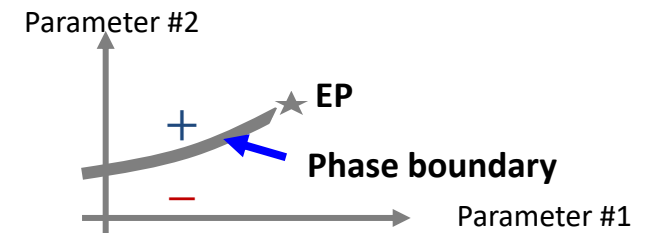
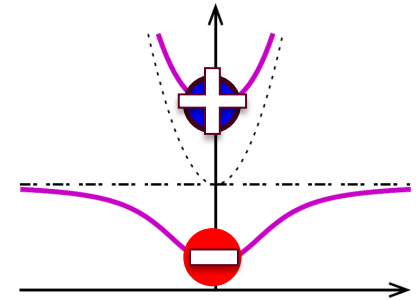
Why a transition?

Non-Hermitian phase transition

A nonequilibrium driven-dissipative electron-hole-photon condensate can be classified into two types

The non-Hermitian nature gives rise to a first-order-like boundary between the types, with an endpoint which is an “exceptional point”

Proposed phase diagram of an electron-hole-photon gas.



Minimal model - two-component damped driven Gross-Pitaevskii equation with noise

$$i\partial_t \Psi_\alpha(\mathbf{r}, t) = \sum_{\beta=l,g} [A_{GP}]_{\alpha\beta} (\nabla^2) \Psi_\beta(\mathbf{r}, t) + \eta_\alpha(\mathbf{r}, t),$$

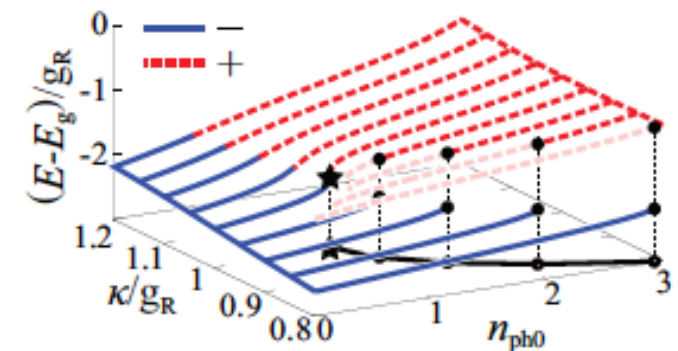
Photon field with dissipation

$$A_{GP}(\nabla^2) = \begin{pmatrix} \omega_l - i\kappa - K_l \nabla^2 & g \\ g & \omega_g + iP - \tilde{K}_g \nabla^2 + \tilde{U}_g |\Psi_g|^2 \end{pmatrix},$$

Exciton field with pumping and nonlinearity

Nontrivial stationary solutions have to be oscillating at a fixed frequency: either E_- or E_+

There is a single point in parameter space where both solutions exist – a so-called exceptional point



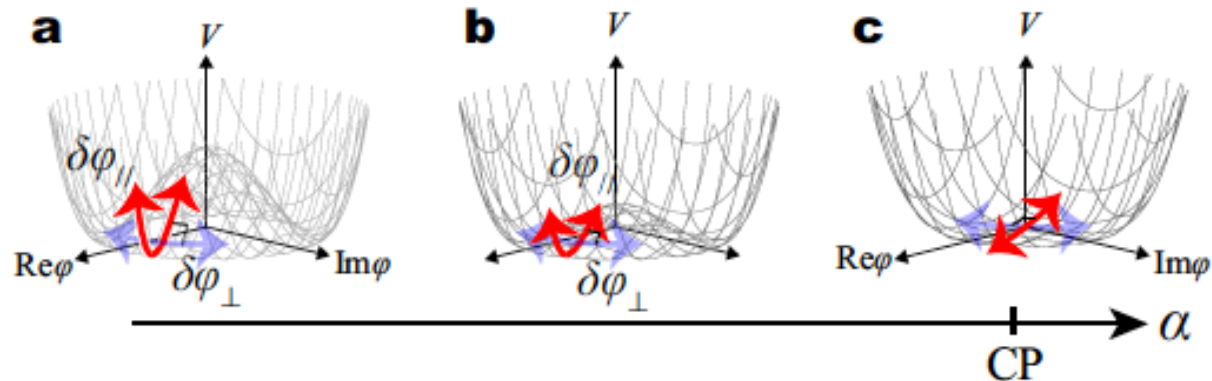
Exceptional point of a non-Hermitian matrix is where it has two identical real eigenvalues

Note: here this is required as a solution of a non-linear GP equation

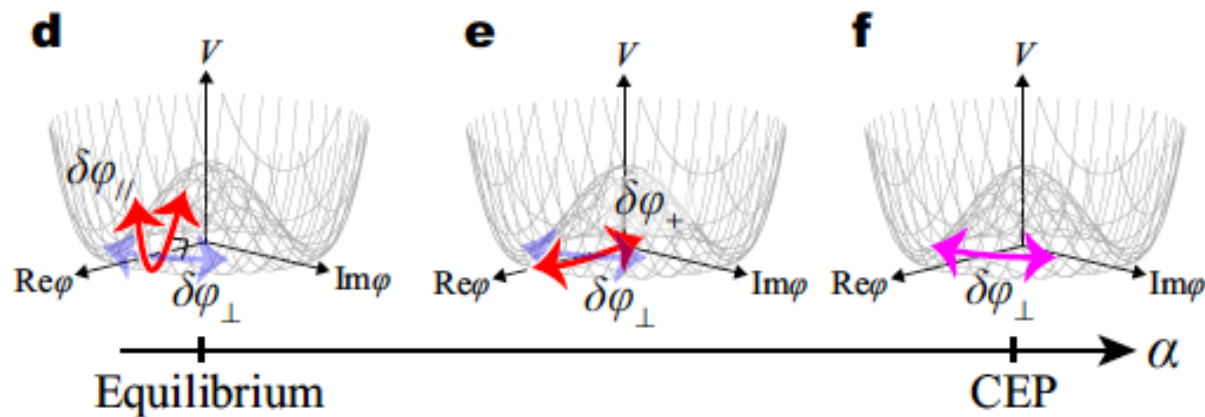
Hanai et al, PRL 2019

Near a critical exceptional point, the two modes coalesce: all fluctuations project onto a massless mode

Conventional critical point (CP)



Critical exceptional point (CEP)



Fluctuations near the EP described by generalized KPZ equation

$$\partial_t \delta\theta_\alpha(\mathbf{r}, t) = \sum_{\beta=1,g} W_{\alpha\beta}(\nabla) \delta\theta_\beta(\mathbf{r}, t) + \sum_{\beta=1,g} \lambda_{\alpha\beta} (\nabla \delta\theta_\beta(\mathbf{r}, t))^2 + \xi_\alpha(\mathbf{r}, t),$$

$$W(\nabla) = \begin{pmatrix} -s_l + \nu_{ll} \nabla^2 & s_l + \nu_{lg} \nabla^2 \\ -s_g + \nu_{gl} \nabla^2 & s_g + \nu_{gg} \nabla^2 \end{pmatrix},$$

IN linear theory, away from criticality, one mode is diffusive, the other is damped

$$\omega_\pm(\mathbf{k}) = \frac{1}{2} \left[-i(\Delta s + Dk^2) \pm \sqrt{-\Delta s^2 + v^2 k^2} \right], \quad \Delta s = s_l - s_g$$

At criticality, both modes become gapless sound modes

$$\omega_\pm(\mathbf{k}) = \pm v|\mathbf{k}| - iDk^2,$$

The two eigenmodes are parallel so the linearized response can only be triangularized

$$\bar{W}(\mathbf{k}) = U(\mathbf{k}) W(\mathbf{k}) U^\dagger(\mathbf{k}) = \begin{pmatrix} -i\omega_-(\mathbf{k}) & \zeta \\ 0 & -i\omega_+(\mathbf{k}) \end{pmatrix}$$

Hanai et al, unpublished

Weak random noise mixes the two fields and gives rise to divergent fluctuations in $d < 6$

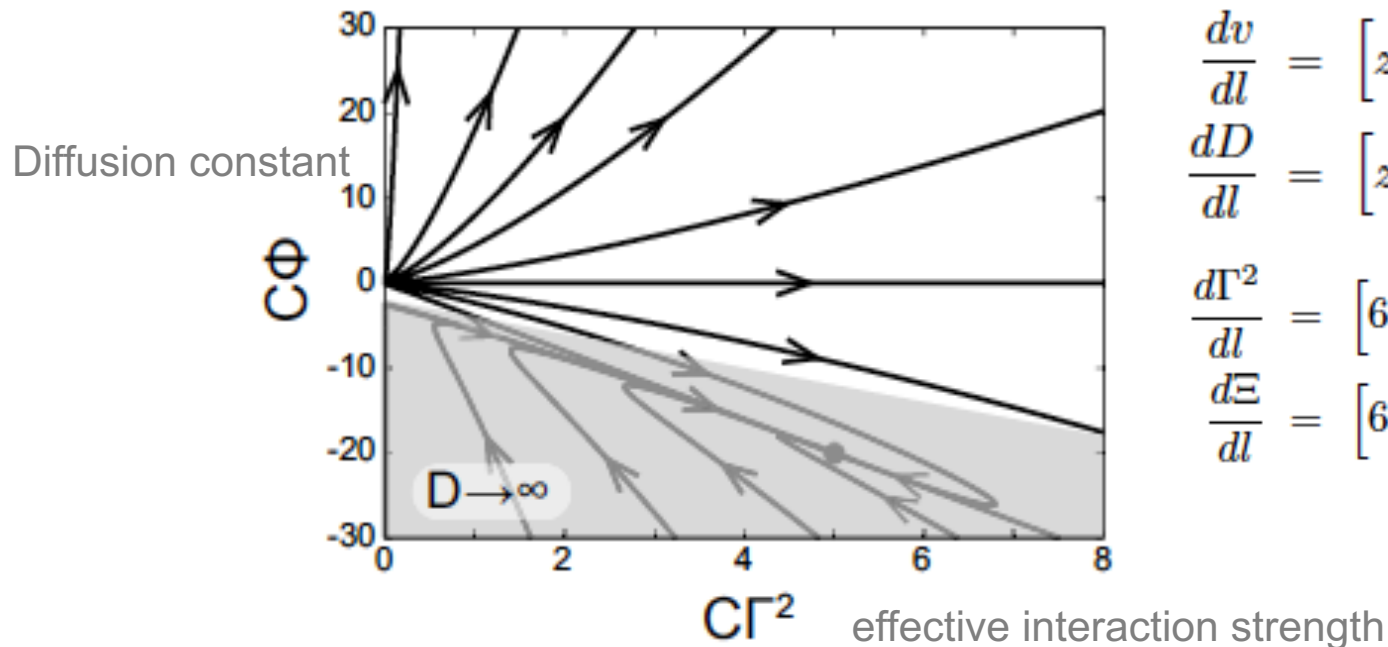
$$\bar{G}^0(\mathbf{k}, \omega) = \begin{pmatrix} \frac{i}{\omega - \omega_-(\mathbf{k})} & \frac{\zeta}{(\omega - \omega_-(\mathbf{k}))(\omega - \omega_+(\mathbf{k}))} \\ 0 & \frac{i}{\omega - \omega_+(\mathbf{k})} \end{pmatrix}.$$

$$\begin{aligned} \langle \delta\theta_\alpha(\mathbf{r}) \delta\theta_\beta(\mathbf{r}') \rangle &\sim \int_0^{\Lambda_c} dk k^{d-1} e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')} \\ &\times \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \bar{G}_{\perp\parallel}^0(\mathbf{k}, \omega) \sigma_{\parallel\parallel} \bar{G}_{\perp\parallel}^0(-\mathbf{k}, -\omega) \\ &= \int_0^{\Lambda_c} dk k^{d-1} e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')} \frac{A}{k^4}, \end{aligned} \quad (5)$$

Perturbative RG

Perturbative RG in $d < 6$ generates an unstable fixed point dividing flows to either a uniform phase, or a strongly disordered phase:

Depends on initial conditions, namely the dominance of photon vs exciton diffusion



$$\frac{dv}{dl} = \left[z - 1 + \frac{C}{d}\Gamma^2 \right] v,$$

$$\frac{dD}{dl} = \left[z - 2 - \frac{C}{2d}\Xi \right] D$$

$$\frac{d\Gamma^2}{dl} = \left[6 - d + \frac{3}{2d}C\Xi + \frac{4+d}{d}C\Gamma^2 \right] \Gamma^2,$$

$$\frac{d\Xi}{dl} = \left[6 - d + \frac{2}{d}C\Xi + \frac{6+d}{d}C\Gamma^2 \right] \Xi.$$

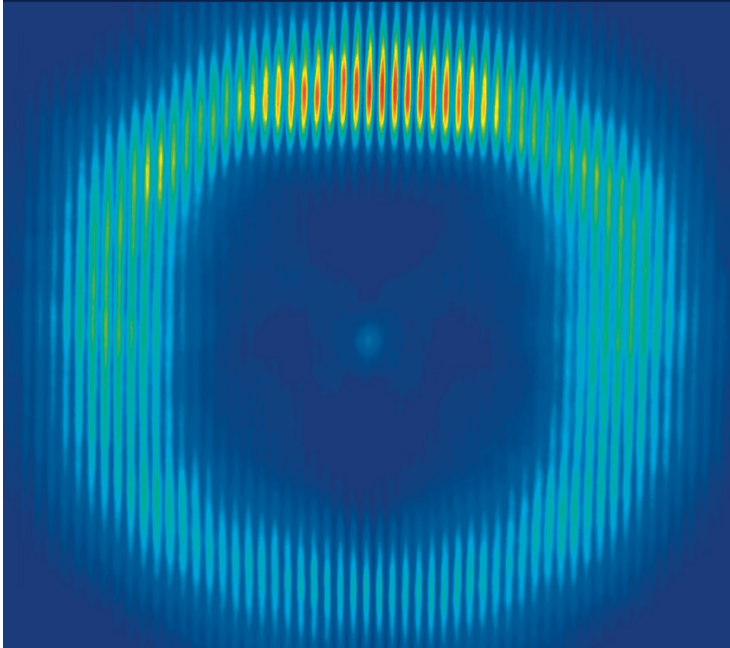
Hanai et al, unpublished

Generalization

- This appears to be a general phenomenon available in principle to any damped-driven 2-component system, not necessarily quantum in origin
 - e.g. an “active” fluid has a local source of energy, which means there can be non-reciprocal forces : $F_{AB} \neq F_{BA}$
 - the simplest version of an exceptional point in a 2 particle system is when $F_{AB} = -F_{BA}$
- We have evidence that we can find this in a 2-component Toner-Tu model (1 component model used to describe bird flocking; 2 components is predator-prey model)
- In Huygens’s coupled clock problem, this is when the in-phase and out-of-phase solutions become dynamically degenerate

UNIVERSAL THEMES OF BOSE-EINSTEIN CONDENSATION

Edited by Nikolaos Proukakis,
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D. W. Snoke, N. P. Proukakis, T. Giamarchi, P. B. Littlewood, T. Greytak, D. Kleppner, W. Ketterle, A. Edelman, A. J. Daley, R. P. Smith, M. J. Davis, T. M. Wright, T. Gasenzer, S. A. Gardiner, T. Langen, J. Schmiedmayer, C. Chin, N. Y. Kim, W. H. Nitsche, Y. Yamamoto, J. Keeling, L. M. Sieberer, E. Altman, L. Chen, S. Diehl, J. Toner, I. Bloch, N. Goldman, N. R. Cooper, J. Dalibard, L. J. LeBlanc, I. B. Spielman, L. Pitaevskii, S. Stringari, N. G. Parker, A. J. Allen, C. F. Barenghi, M. Ueda, J. Klaers, M. Weitz, A. Chiocchetta, A. Gambassi, I. Carusotto, D. N. Krizhanovskii, K. Guda, M. Sich, M. S. Skolnick, L. Dominici, D. Sanvitto, G. Christmann, P. G. Savvidis, J. J. Baumberg, P. R. Eastham, B. Rosenow, H. Terças, D. D. Solnyshkov, G. Malpuech, H. Salman, N. G. Berloff, S. O. Demokritov, R. A. Duine, A. Brataas, S. A. Bender, Y. Tserkovnyak, H. Chen, A. H. MacDonald, C. Kollath, C. Rüegg, C. J. Pethick, T. Schäfer, A. Schwenk, G. R. Pickett, N. Banik, P. Sikivie, G. Dvali, C. Gomez