

Polaron mobility in the “beyond quasiparticle” regime

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[arXiv:1812.10336](https://arxiv.org/abs/1812.10336) → PRL

Outline:

A conceptual question, and a puzzle ...

Classical Eq. of motion with **friction** (translation invariant system, linear response)

$$M \ddot{R} + \eta \dot{R} = F \quad \text{or} \quad M[-\omega^2 - i\omega / \tau] R_\omega = F_\omega$$

What are quantum mechanical consequences of formally having $\hbar / \tau \gg T$, say as $T \rightarrow 0$?

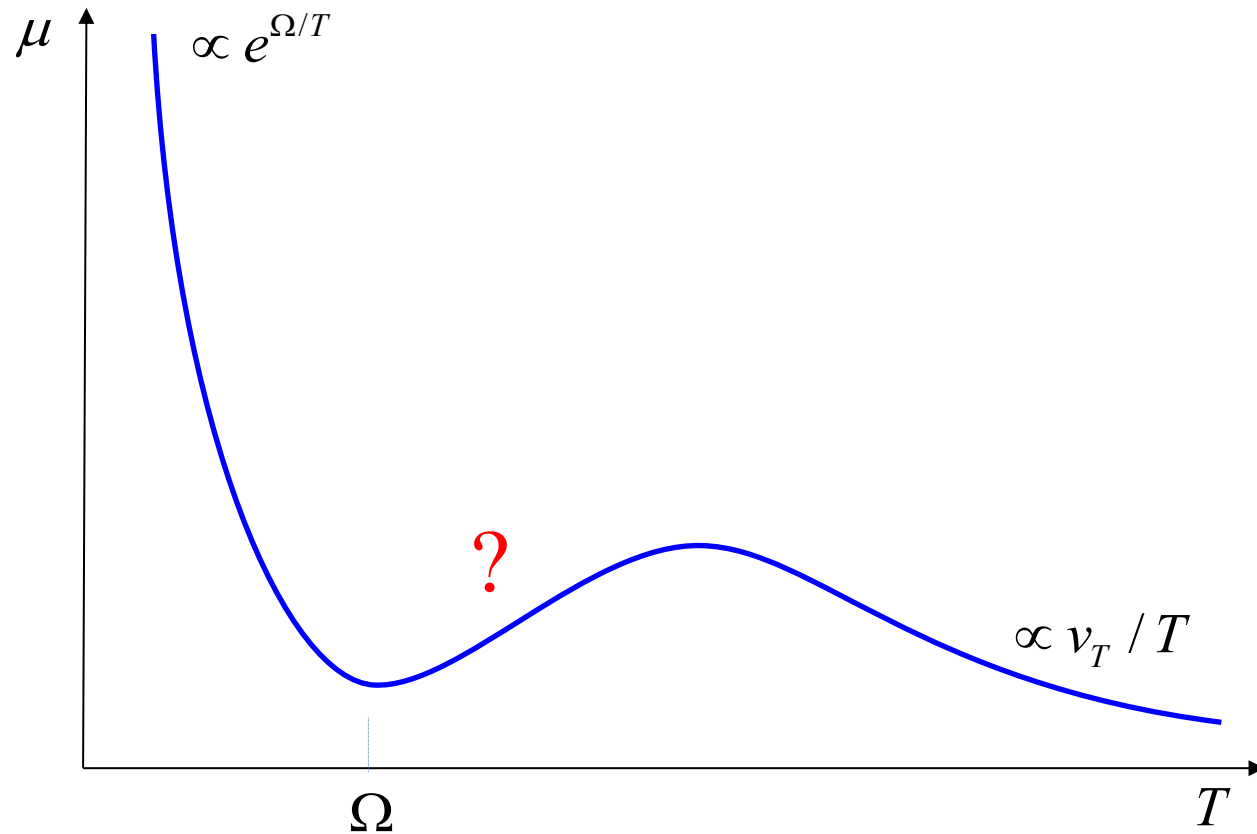
Is it possible in principle? What about $E\tau \geq \hbar$?

Does this “quantum reading” makes sense?

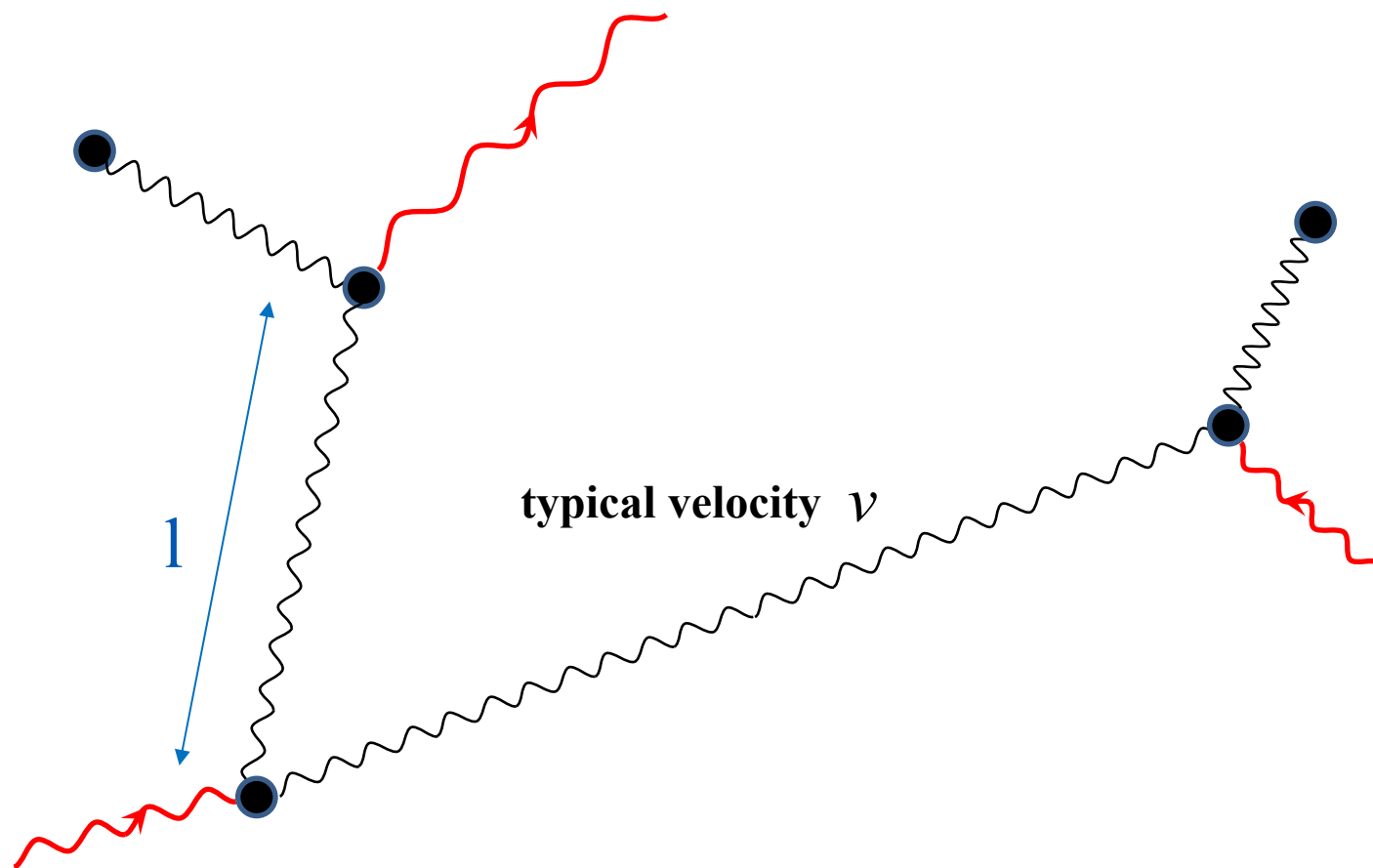
$$-\omega^2 M \underbrace{[1 + i / \omega \tau]}_{\text{mass renormalization with diverging } M_*} R_\omega = F$$

mass renormalization with **diverging** M_* at low ω or T

Electrons scattered by optical phonons with frequency Ω . Number of phonons $N_{ph} = \frac{1}{e^{\Omega/T} - 1}$



Mobility minimum: moving faster despite increasing N_{ph}



Drude model

$$\mathbf{\dot{p}} = F - p / \tau \quad (\text{momentum relaxation})$$

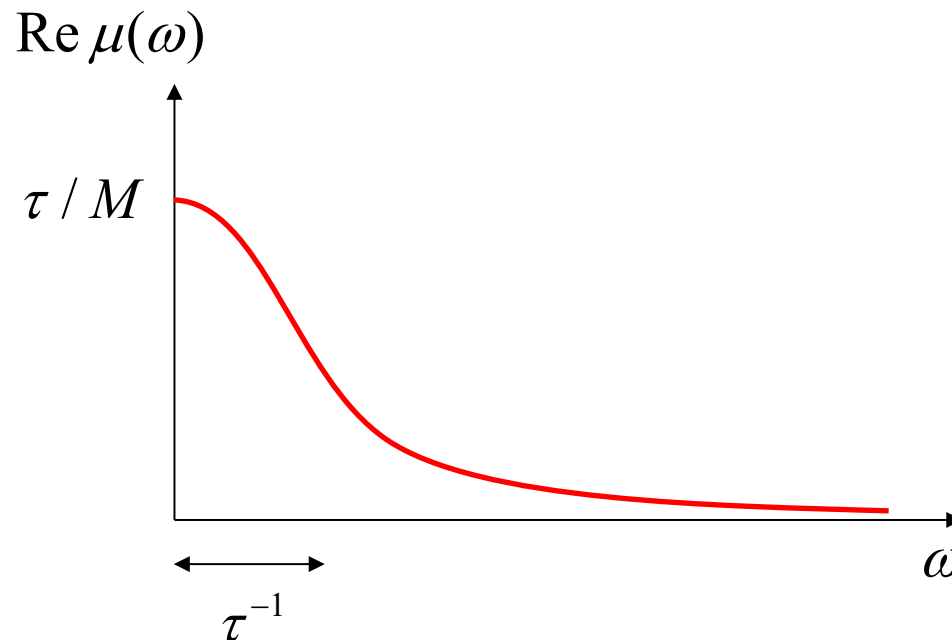
$$\mu = v / F = p / MF \quad (\text{mobility})$$



$$\mu(\omega) = \frac{\tau}{M(1 - i\omega\tau)}$$

Static mobility: $\mu = \tau / M \rightarrow$ impossible to separate transport scattering time from mass
which **has to** be renormalized in an interacting system $M \rightarrow M_*$

Frequency dependence: Half-width criterion based on $\text{Re } \mu(\omega) = \frac{\mu}{1 + (\omega\tau)^2}$ appears to provide
a direct measure of τ^{-1}



Drude model

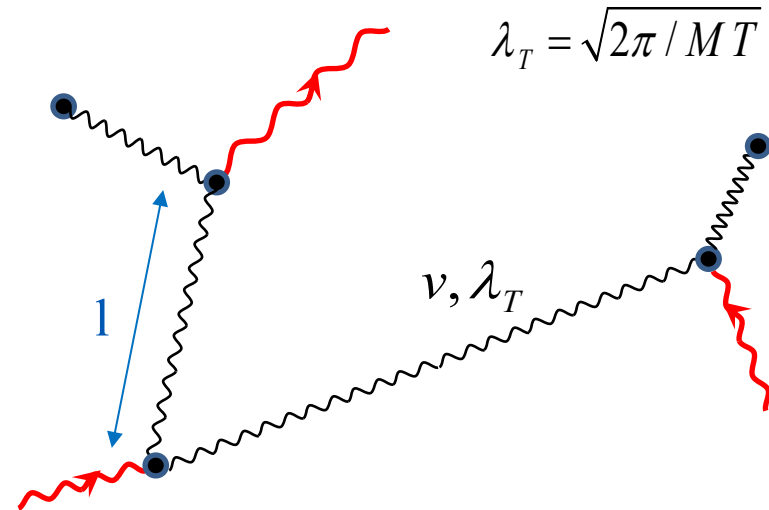
When does the conventional wisdom and the kinetic equation approach work?

(i) **Rare collisions** with ballistic propagation in between,

$$l \gg \lambda_T \quad (\text{Mott-Ioffe-Riegel criterion})$$

(ii) To define thermal energy $\sim T$ requires time scales

$$\tau > \hbar / T \quad (\text{Planckian bound})$$

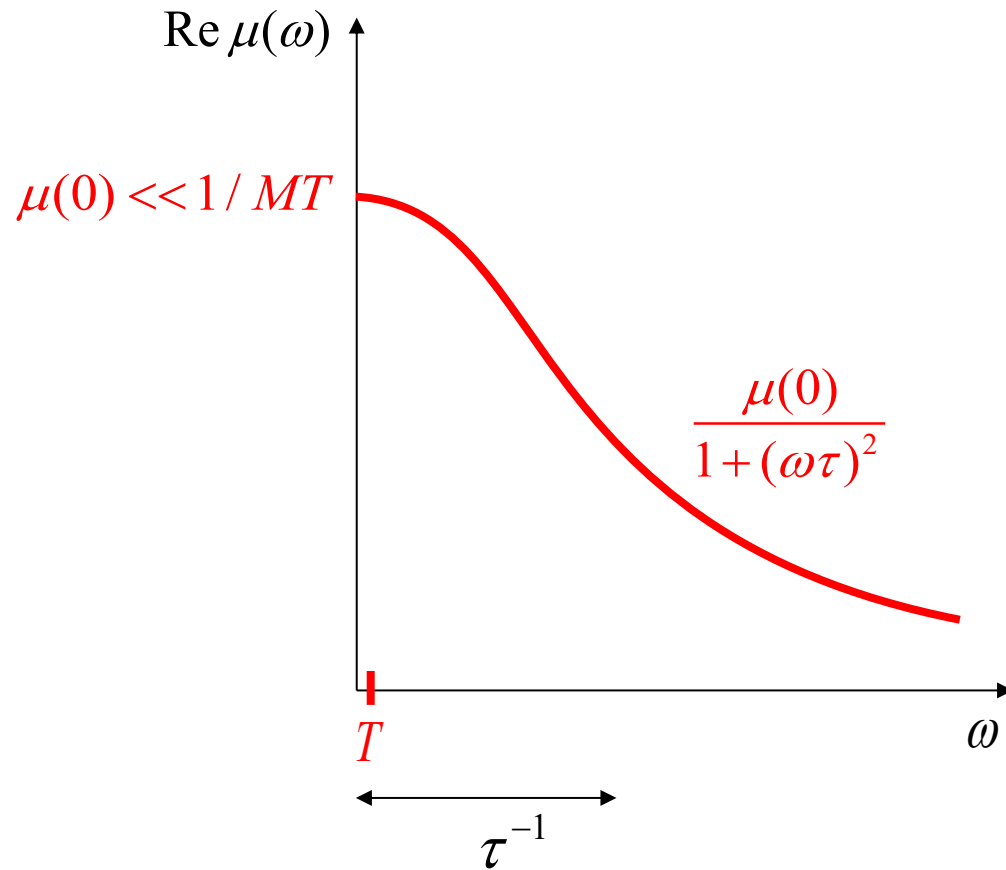


The two conditions/arguments are consistent ($v\tau = l$) $\gg$ ($v / T = \lambda$),

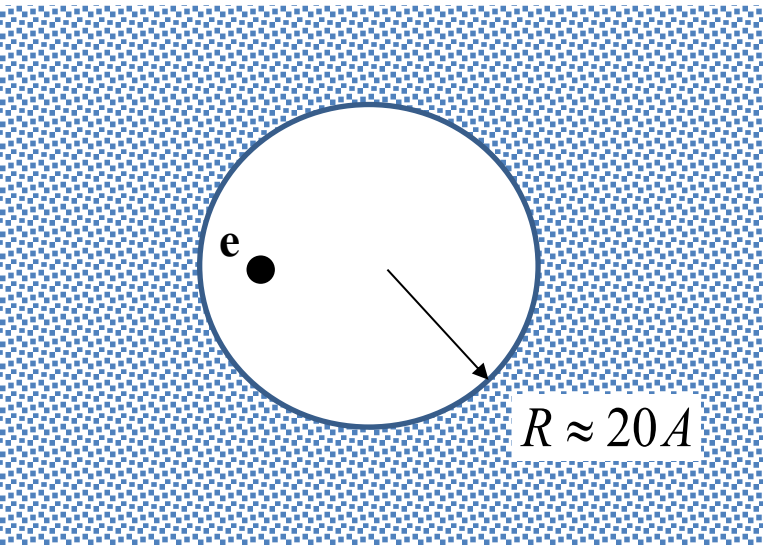
and imply that the mobility is “large”: $\mu \gg 1 / MT$

What if these conditions are not satisfied ?

Can this be even seen experimentally?

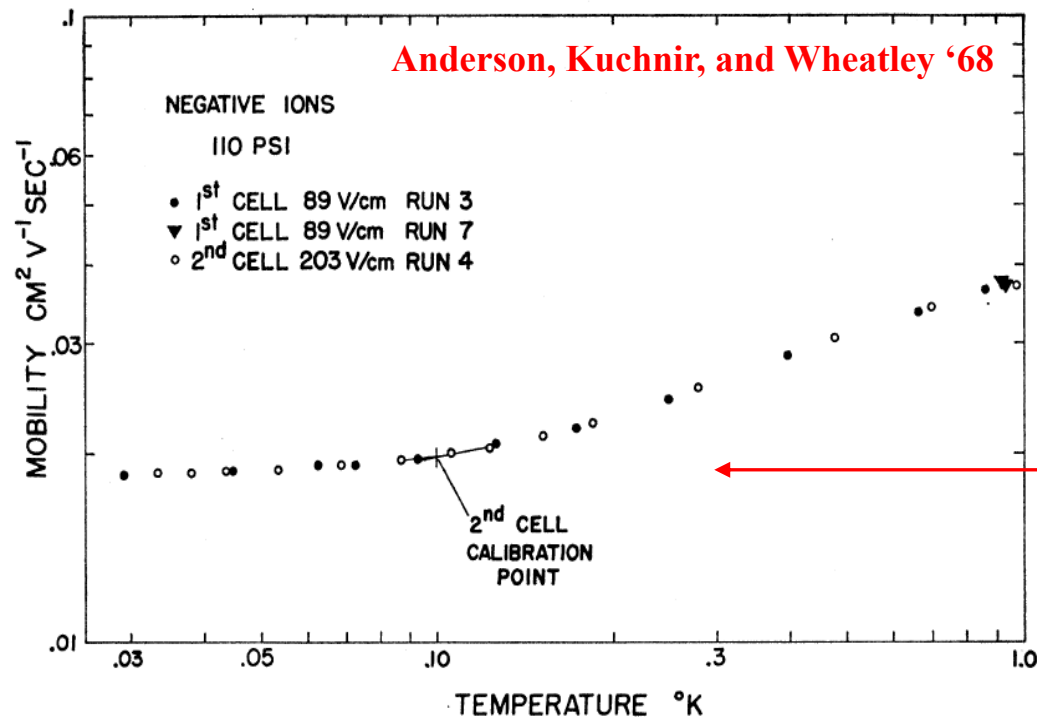


Electron Bubbles in He-3



$$M \approx 200 m_3 \quad (\text{large excluded volume})$$

$$g = \frac{\sigma p_F^2}{3\pi^2} \gg 1 \quad (\text{about } \mathbf{25} \text{ at } P=0)$$



Mobility in the Fermi liquid regime $T \ll E_F$
is temperature independent!

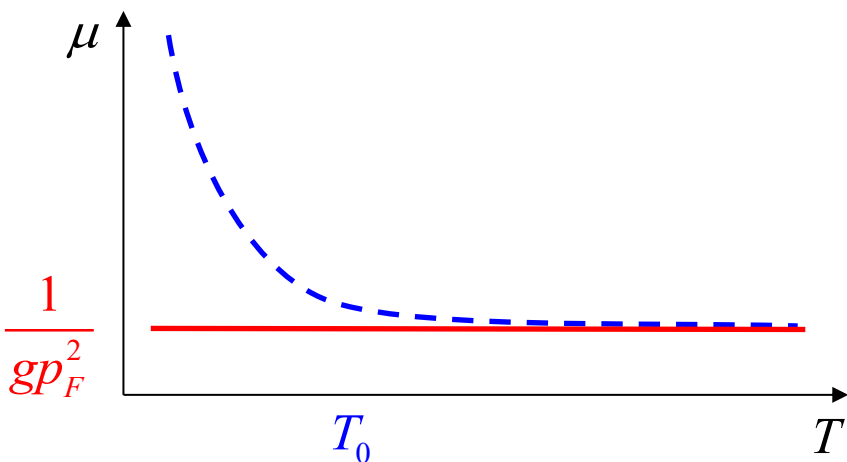
Electron Bubbles in He-3

At high temperature $\mu(0)$ can be explained by

$$\tau^{-1} = g \frac{p_F^2}{M}$$

At $T < T_0 = p_F^2 / M$ the recoil energy cannot be neglected and it is expected that $\mu \propto T^{-2}$.

Not observed!



- Before T_0 is reached, we already face $\tau^{-1} = gT_0 \gg T_0$

- For $g \gg 1$ one can show rigorously that $\mu(\omega) = \frac{\tau}{M(1-i\omega\tau)}$

behavior will continue down to $T \sim T_0 e^{-g}$

The Planckian bound and the MIR criterion are formally strongly violated. Drude survives!
NP '90-93

Classically, this is overdamped motion → bubbles placed in the harmonic potential will not oscillate

Overdamped motion is OK classically but in many respects makes no sense quantum mechanically!

Ohmic dissipation

Start with the classical Eq. of motion with friction, $M \ddot{R} + M \dot{R} / \tau = F$,

and develop a quantum model with this semiclassical behavior.

Magalinskii 1959, Feynman-Vernon 1963, Caldeira-Leggett 1980-83

Particle coupled to a set of harmonic oscillators $\sum_{\alpha} g_{\alpha} R \cdot x_{\alpha}$

Quantum version (through action): $S = \frac{T}{2} \sum_m M \left\{ \omega_m^2 + \tau^{-1} |\omega_m| \right\} |R_m|^2$, $Z = \oint e^{-S} DR_{\tau}$
($\omega_m = 2\pi T m$)

↓

$$J(\omega) = \frac{\pi}{2} \sum_{\alpha} \frac{g_{\alpha}^2}{m_{\alpha} \omega_{\alpha}} \delta(\omega - \omega_{\alpha}) = \text{(Ohmic case)} = \frac{M}{\tau} \omega$$

Ohmic dissipation: Particle coupled to a set of harmonic oscillators $\sum_{\alpha} g_{\alpha} R \cdot x_{\alpha}$

For quadratic action $S = \frac{T}{2} \sum_m M \left\{ \omega_m^2 + \tau^{-1} \left| \omega_m \right| \right\} \left| R_m \right|^2$ calculations are exact



$$\mu(\omega) = \frac{\tau}{M(1 - i\omega\tau)} \quad (\text{including } \tau T = 1)$$

Alternative “reading” when friction is large:

$$S \equiv \frac{T}{2} \sum_m \underset{\text{mass renormalization}}{M \left[1 + 1 / \tau \left| \omega_m \right| \right]} \omega_m^2 \left| R_m \right|^2$$

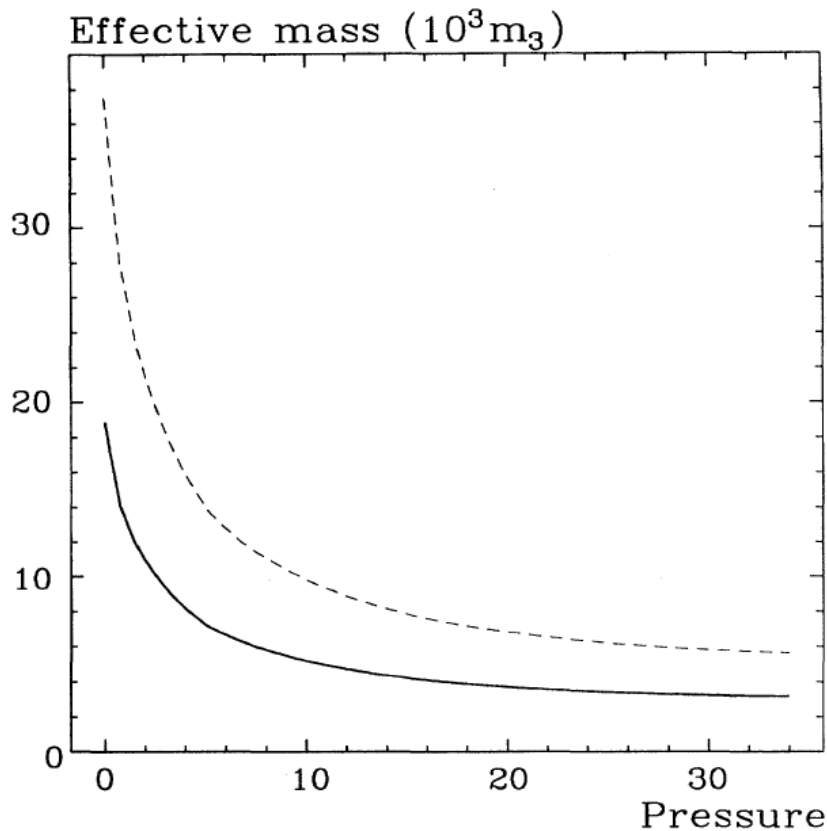
- Effective mass diverges as $M_* = M g \frac{T_0}{T} \equiv \frac{M}{T\tau}$ (for bubbles, recoil can be neglected: $p_F^2 / M_* = T / g$)
- **Insist on Planckian bound** for the scattering time $\tau_* \sim 1 / T$; Note that $\mu = \frac{\tau_*}{M_*} = \frac{\tau}{M}$
- Half-width in frequency is NOT defining scattering time; it is rather mass undressing effect

Electron Bubbles in He-3

are an Ohmic dissipative system above T_c

Proof of alternative reading: In superfluid $^3\text{He-B}$, the ballistic motion should see

$$M_* \sim Mg \frac{T_0}{\Delta_C} ? \quad M$$



About $20,000 m_3$ at zero pressure

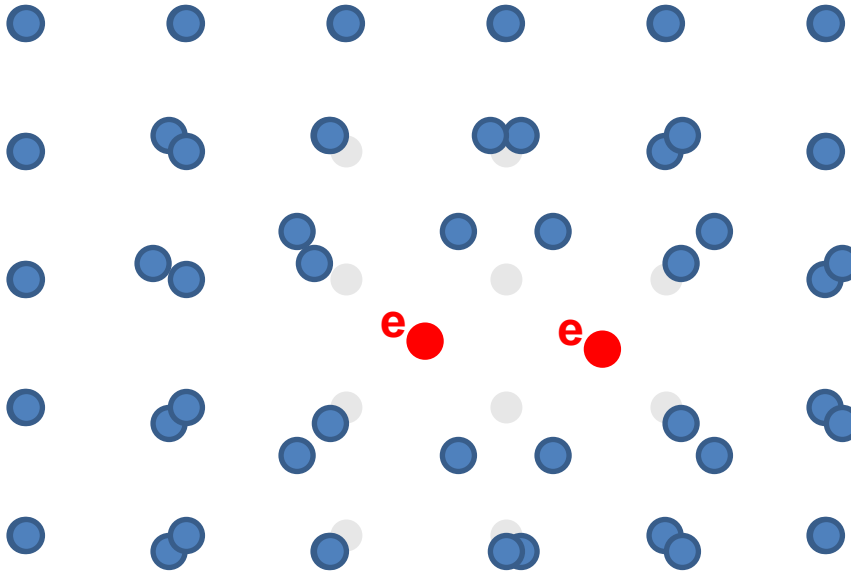
If not superfluidity it would increase to

$$200 m_3 \times g \times e^g \sim 10^{13} m_3$$

Frohlich polaron problem: $H = H_{\text{particle}} + H_{\text{environment}} + H_{\text{coupling}} \rightarrow$

$E(p), M^*, G(p, t), \mu$
quasiparticle

Electrons in semiconducting crystals (electron-phonon polarons)



$$H = \sum_p \varepsilon(p) a_p^\dagger a_p + \quad \text{electron}$$

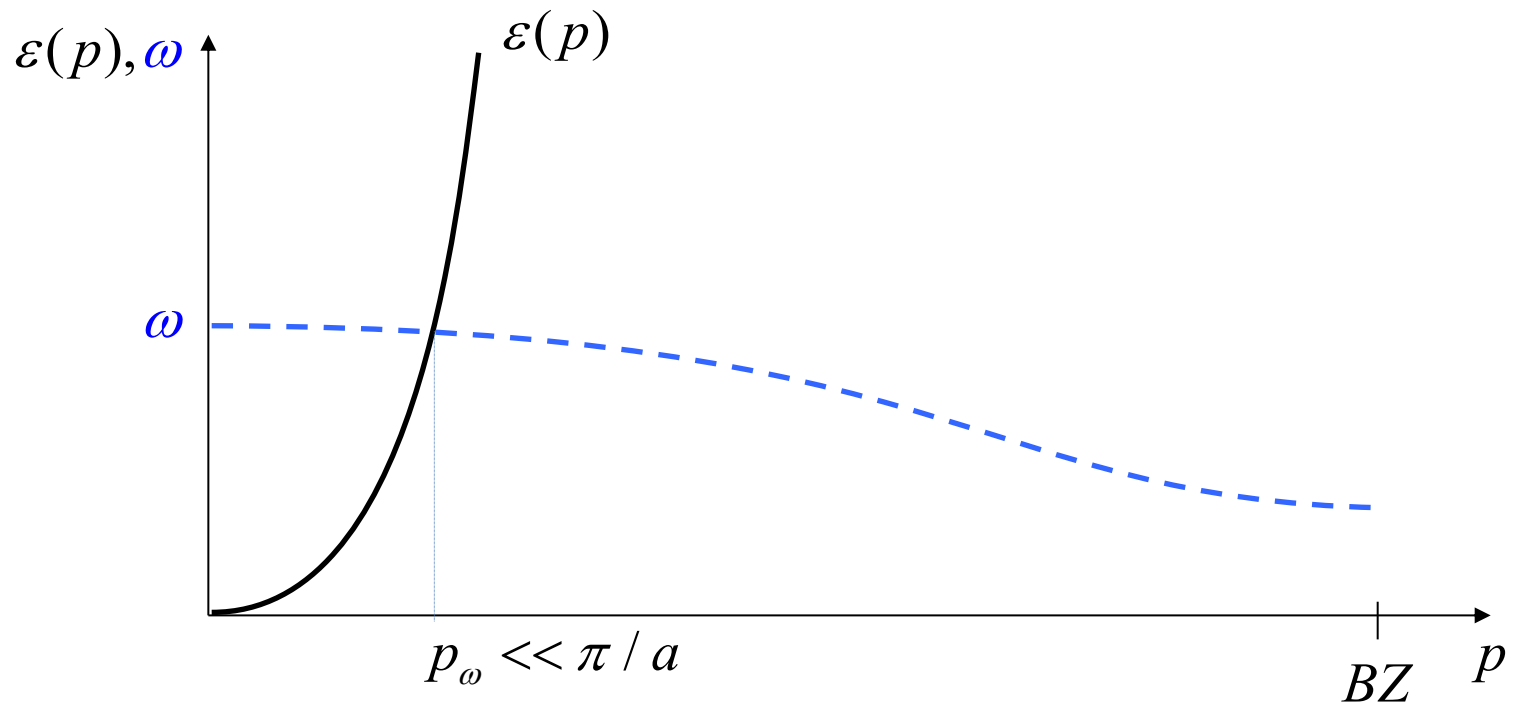
$$\sum_q \omega(q) (b_q^\dagger b_q + 1/2) + \quad \text{phonons}$$

$$\sum_{pq} \left(V_q a_{p-q}^\dagger a_p b_q^\dagger + h.c. \right) \quad \text{el.-ph. interaction}$$

In the non-degenerate (dilute gas) limit

$$\varepsilon(p) = \frac{p^2}{2M}, \quad \omega(q) = \Omega = \text{const}, \quad V(q) = \alpha \frac{2\sqrt{2}\pi \Omega^{3/2}}{M^{1/2} q^2}$$

Electron charge – displacement dipole coupling



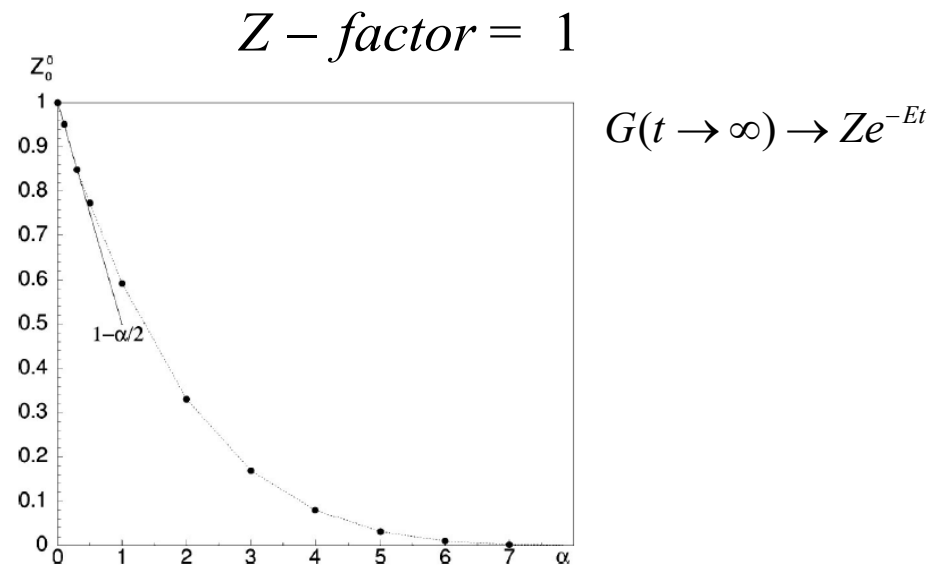
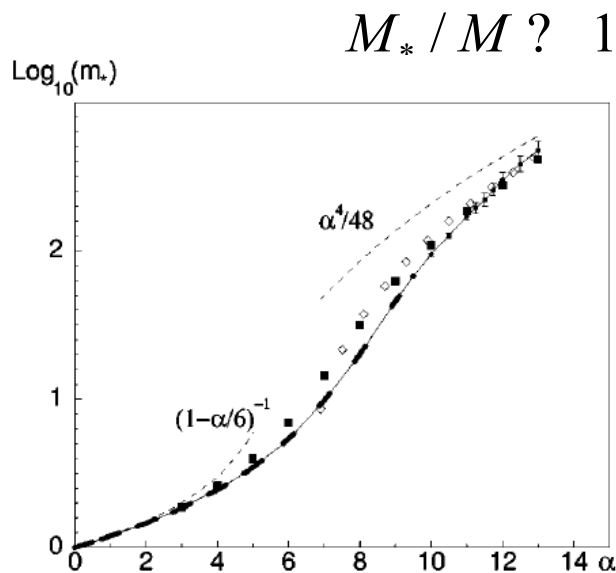
Weak-coupling ($\alpha \ll 1$), $T \ll \Omega$: perturbation theory [D.C. Langreth, L.P. Kadanoff PR '63,64]

$$\mu = \frac{1}{2M_*\alpha\Omega} e^{\Omega/T} \quad \text{with} \quad M_* \approx M / (1 - \alpha/6)$$

High temperature $T \ll \Omega$: quasielastic scattering + Migdal's theorem

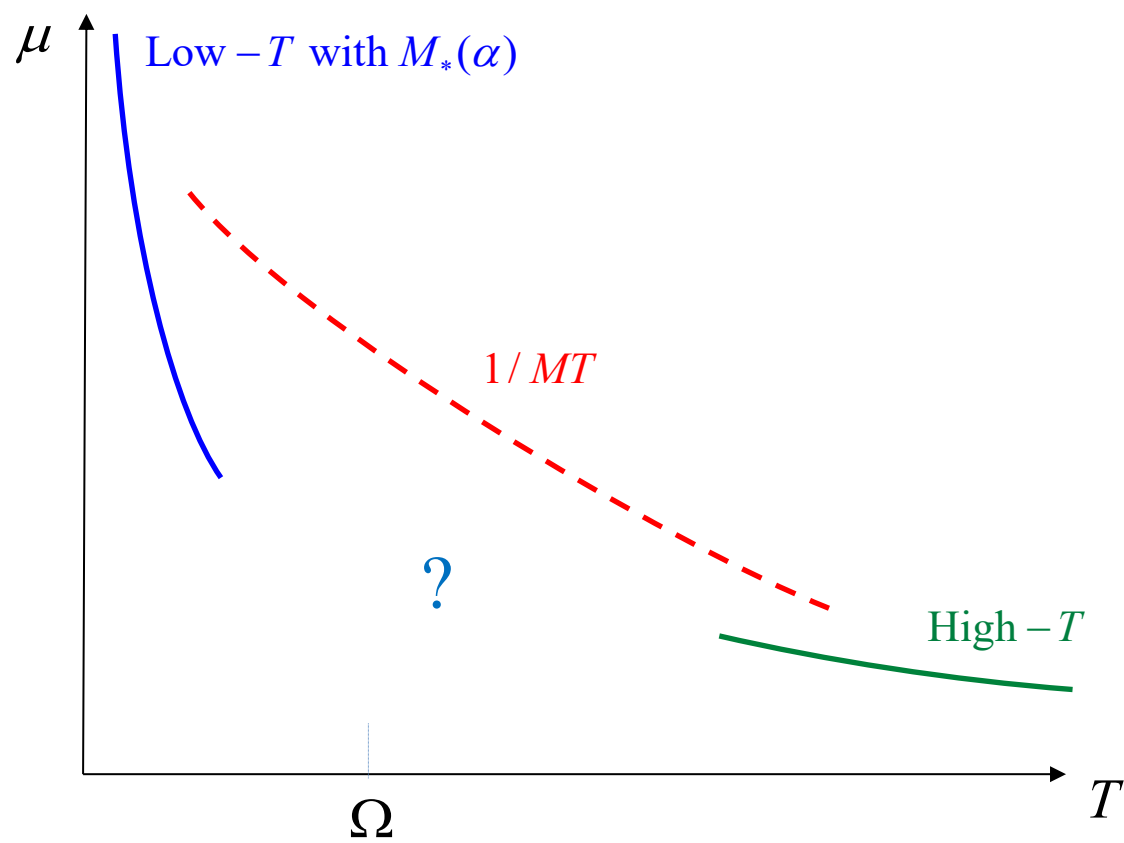
$$\mu = \frac{1}{3\sqrt{\pi}M\alpha\sqrt{\Omega T}}$$

For $\alpha > 1$ perturbation theory fails:



[A.S. Mishchenko, A. Sakamoto, NP, B. Svistunov PRB '2000]

and the Plankian bound appears to be violated



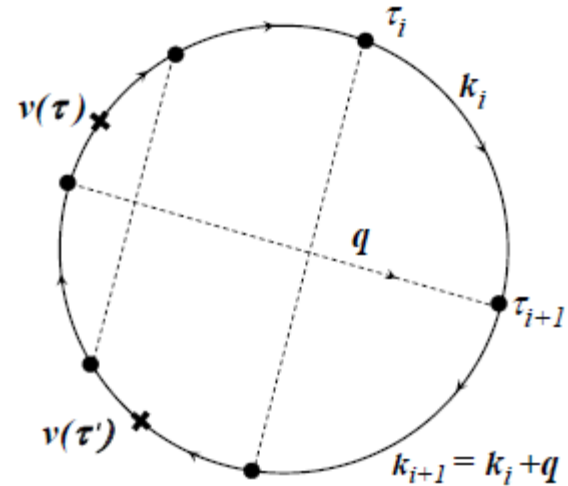
Solution by Diagrammatic Monte Carlo (Diag.MC) for any value of coupling constant

Diag.MC = unbiased MC sampling of all contributing Feynman diagrams for quantities of interest

Here, diagrams for current-current correlation function $C(\tau)$

$$C_m = \int_0^\beta d\tau e^{i\omega_m \tau} C(\tau) = \frac{\langle |k_m|^2 \rangle}{3\beta M^2}; \quad k_m = \sum_i k_i \frac{e^{i\omega_m \tau_{i+1}} - e^{i\omega_m \tau_i}}{i\omega_m}$$

(A given diagram's contribution to correlator in Matsubara-frequency space)

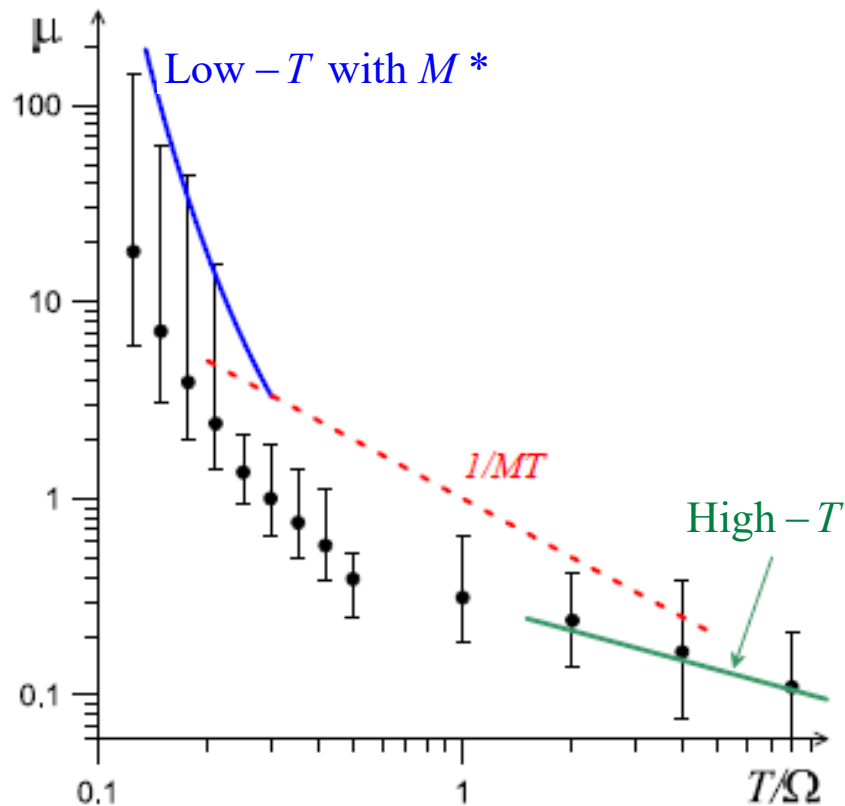


- Statistical error bars in Matsubara space: at the level of six significant digits

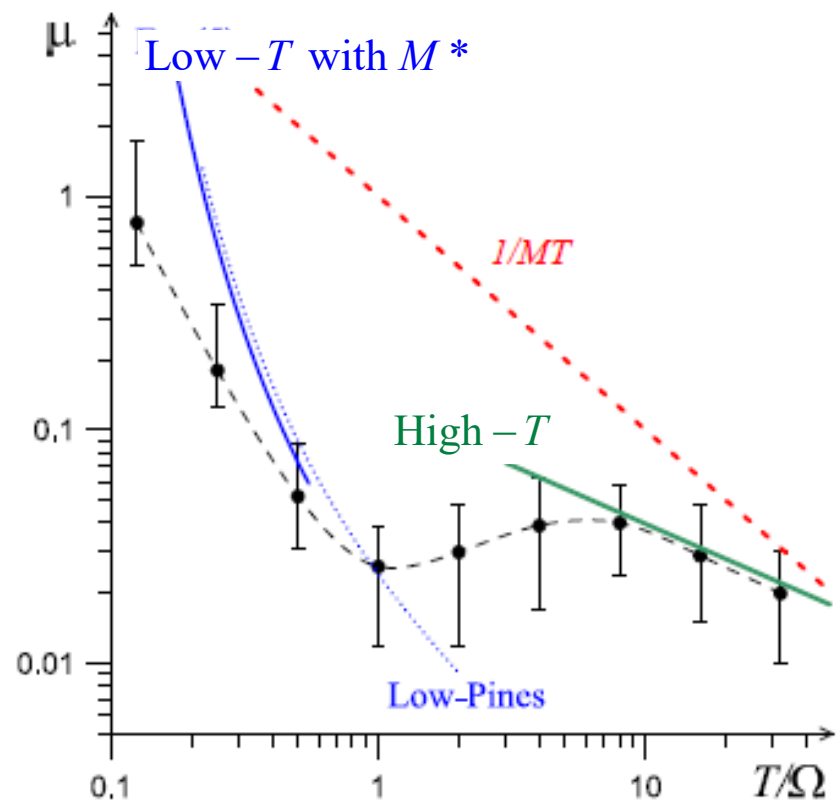
- Systematic errors: None

+ numeric analytic continuation to real frequency

Error bars on the real-frequency axis are always conditional and can be rather large.



$$\alpha = 2.5$$



$$\alpha = 6$$

Mott-Ioffe-Riegel criterion is strongly violated

Intermediate coupling $\rightarrow$ “flat” behavior at $T \sim \Omega$

Strong coupling $\rightarrow$ **mobility minimum** at $T \sim \Omega$
despite decreasing number of thermal phonons!

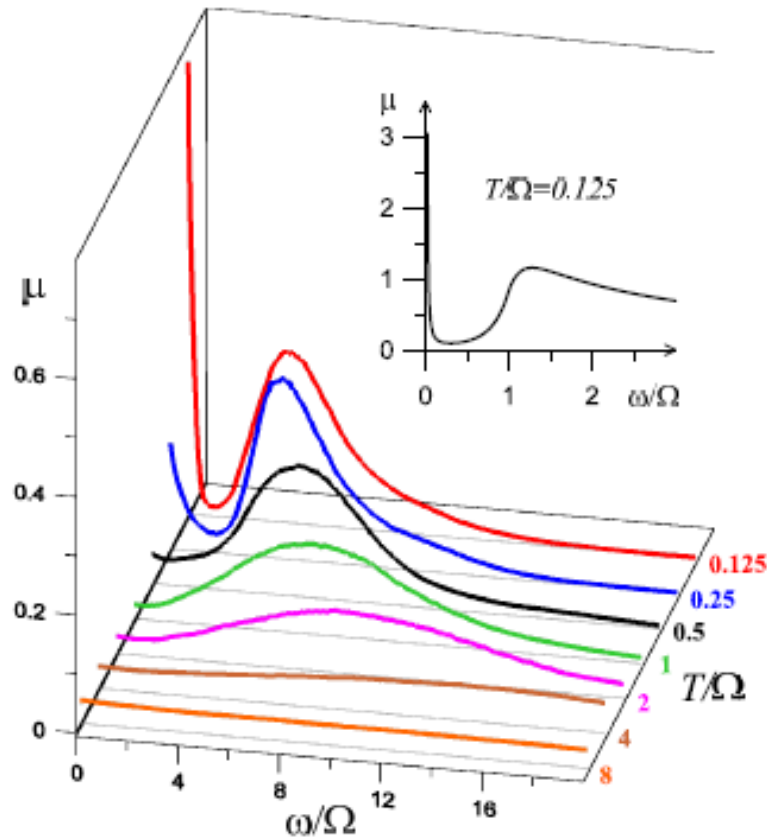


As temperature is lowered,
strong effective mass renormalization
effects are more important than

$$N_{\text{BOSE}} = \frac{1}{e^{\Omega/T} - 1}$$

The Plankian bound condition implies $M^*(T) : \frac{1}{\mu(T)T}$

If energy/frequency dependent mass renormalization is crucial, then $\mu(\omega)$ may have unconventional properties too



No Drude peak for $0.5 < T / \Omega < 4$,
minimum, not maximum at zero frequency!

Conclusions:

Whenever the Plankian bound on inelastic transport scattering time ($\tau^{-1} < T$) is violated,

- expect strong temperature/frequency dependent mass renormalization**

- frequency dependence of $\mu(\omega)$ cannot be used to define τ^{-1}**

- non-monotonic $\mu(T)$ dependence around $T \sim \Omega$**

Spectral density $J(\omega) = \frac{\pi}{2} \sum_{\alpha} \frac{g_{\alpha}^2}{m_{\alpha} \omega_{\alpha}} \delta(\omega - \omega_{\alpha})$

Ohmic case: $J(\omega) = \frac{M}{\tau} \omega$

$$J(\omega) = \frac{\pi}{2} \sum_{\alpha} \frac{g_{\alpha}^2}{m_{\alpha} \omega_{\alpha}} \delta(\omega - \omega_{\alpha}) = (\text{Ohmic case}) = \frac{M}{\tau} \omega$$

A. I. Ahonen, J. Koko, M. A. Paalanen, R. C. Richardson, W. Schoepe, and Y. Takano, J. Low Temp. Phys. 30, 205 (1978).

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