

The unitary Fermi gas

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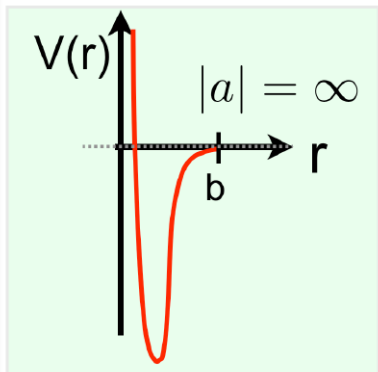
- K. Van Houcke, FW, T. Ohgoe, N. Prokof'ev, B. Svistunov,
Diagrammatic Monte Carlo algorithm for the resonant Fermi gas, PRB 2019
- R. Rossi, T. Ohgoe, K. Van Houcke, FW,
*Resummation of diagrammatic series with zero convergence radius
for strongly correlated fermions*, PRL 2018
- R. Rossi, T. Ohgoe, E. Kozik, N. Prokof'ev, B. Svistunov, K. Van Houcke, FW,
Contact and Momentum Distribution of the Unitary Fermi Gas, PRL 2018
- T. Ohgoe, R. Rossi, K. Van Houcke, FW,
work in progress

NOTE: this is an edited version of the slides
Unpublished work was removed

Unitary Fermi gas

Spin- $\frac{1}{2}$ fermions, 3D **continuous** space, interactions $\left\{ \begin{array}{l} \text{zero range} \\ \text{scattering length } a = \infty \end{array} \right.$

Universality hypothesis:



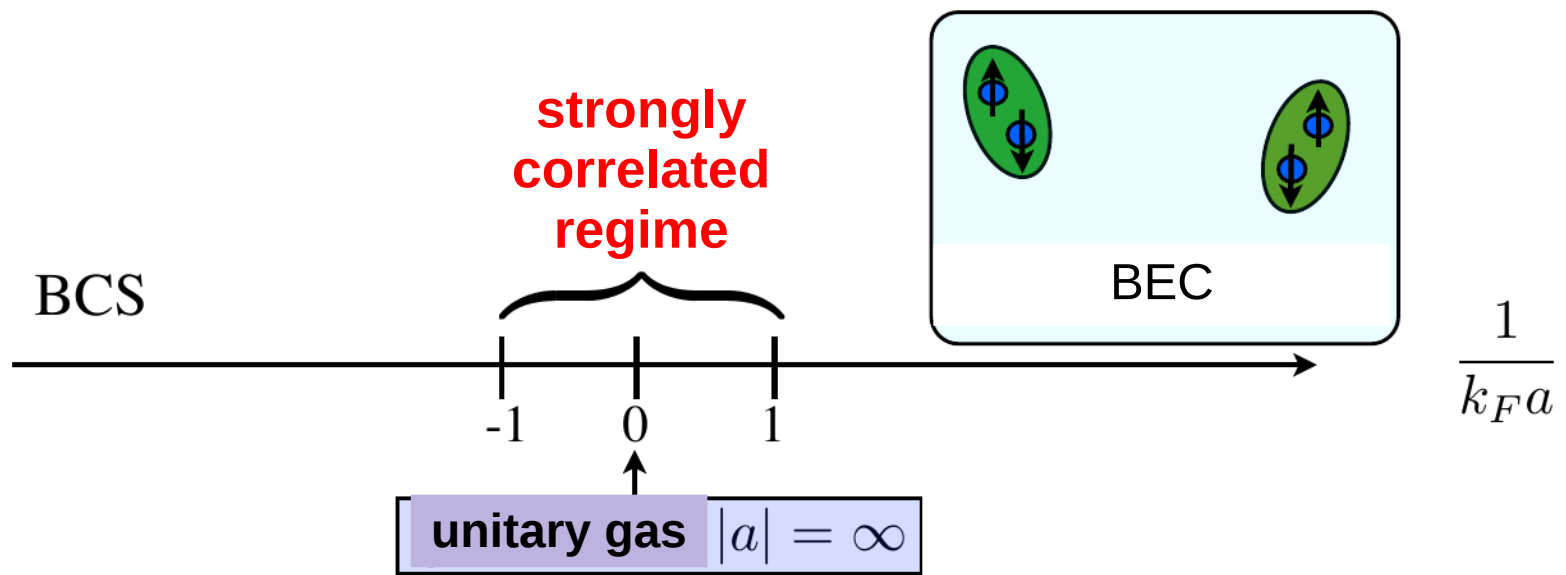
Zero-range limit: $\left\{ \begin{array}{l} n^{-1/3} \gg b \\ \lambda \gg b \end{array} \right. \quad \lambda \equiv \sqrt{\frac{2\pi\hbar^2}{mk_B T}}$

$\Rightarrow$ Properties do not depend on $V(r)$

$(N_\uparrow = N_\downarrow) \quad n(T, \mu)\lambda^3 = \text{universal function of } \beta\mu$

Construction from Hubbard model:

- $\frac{U}{t} = -7.913552\dots$ (appearance of 2-body bound state)
- thermodynamic limit
- filling $\rightarrow 0$ with $\frac{T}{T_F}$ fixed (= continuum limit)

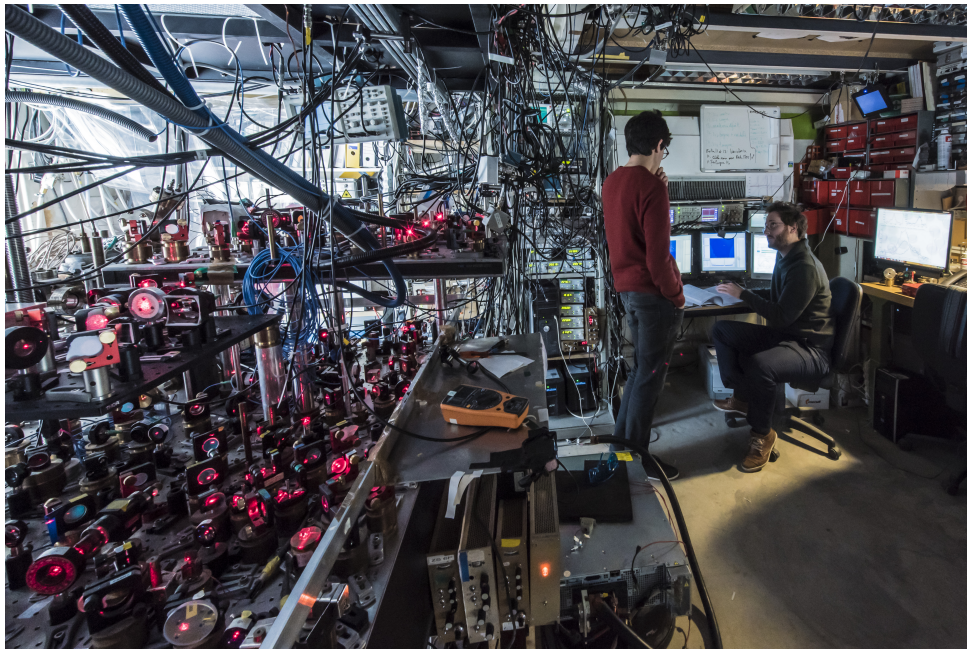


cold atom experiments

fermionic atom, 2 internal states, Feshbach resonance

←→
accurate
comparison

zero-range theory



*also relevant
in neutron stars*

$$\begin{aligned}
 \overrightarrow{G} &= \overrightarrow{G^0} + \overrightarrow{G^0} \text{ (self-energy loop) } + \overrightarrow{G^0} \text{ (two self-energy loops) } + \overrightarrow{G^0} \text{ (three self-energy loops) } + \dots \\
 &+ \overrightarrow{G^0} \text{ (bubble) } + \overrightarrow{G^0} \text{ (two bubbles) } + \overrightarrow{G^0} \text{ (three bubbles) } + \dots
 \end{aligned}$$

Ladder summation:

$$\overrightarrow{\Gamma^0} = \bullet + \text{ (bubble) } + \text{ (two bubbles) } + \dots$$

$\Rightarrow \Gamma^0$ is well-defined in the continuum limit, which can be taken analytically

Dyson equation:

$$\overrightarrow{G} = \overrightarrow{G^0} + \overrightarrow{G^0} \Sigma \overrightarrow{G^0} + \overrightarrow{G^0} \Sigma \overrightarrow{G^0} \Sigma \overrightarrow{G^0} + \dots$$

Self-energy:

$$\Sigma = \text{ (self-energy loop) } + \text{ (two self-energy loops) } + \text{ (three self-energy loops) } + \dots$$

$$\begin{aligned}
 \overrightarrow{G} &= \overrightarrow{G^0} + \overrightarrow{G^0} \text{ (loop) } + \overrightarrow{G^0} \text{ (2 loops) } + \overrightarrow{G^0} \text{ (3 loops) } + \overrightarrow{G^0} \text{ (4 loops) } + \dots
 \end{aligned}$$

Ladder summation:

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Dyson equation:

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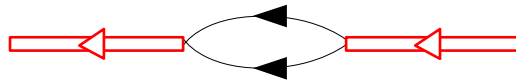
Self-energy:

$$\Sigma = \text{ (loop) } + \text{ (2 loops) } + \text{ (3 loops) } + \dots$$

sum all diagrams up to order ~9
by Diag MC

(“ladder scheme”)

avoid double-counting:



forbidden

$$Q = \sum_{n=0}^{\infty} a_n$$

$$a_1 = \text{[diagram of a single loop with two external lines]}$$

Gaussian pair fluctuation

$$a_9 = \text{[diagram of a complex multi-loop structure]} + \dots$$

Problem: *zero convergence radius*

$$Q \stackrel{?}{=} \sum_{n=0}^{\infty} a_n$$

$$a_1 = \text{[diagram of a single loop with two external legs]}$$

$\Leftrightarrow$ Gaussian pair fluctuation

$$a_9 = \text{[diagram of a complex multi-loop structure]} + \dots$$

Problem: *zero convergence radius*

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Problem: *zero convergence radius*

Solution:

construct $Q(z)$ /
$$\begin{cases} \text{Taylor}[Q(z)]_{z \rightarrow 0+} \hat{=} \sum_{n=0}^{\infty} a_n z^n \\ Q(z=1) = Q_{\text{phys}} \end{cases}$$

$\{a_n\}$
 $\xrightarrow{\text{resummation}}$
 $Q(1)$

$$Q \stackrel{?}{=} \sum_{n=0}^{\infty} a_n$$

$$a_1 = \text{[diagram of a single instanton: a rectangle with a semi-circle on top and arrows on the horizontal lines]}$$

$\Leftrightarrow$ Gaussian pair fluctuation

$$a_9 = \text{[diagram of a complex multi-instanton configuration with many loops and arrows]} + \dots$$

Problem: *zero convergence radius*

Solution:

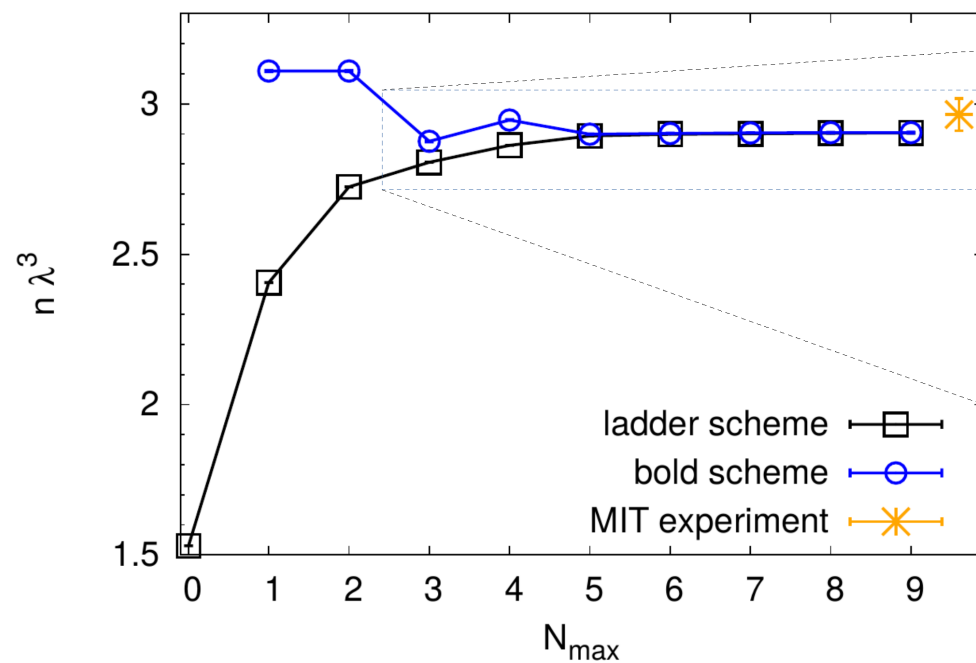
$$\text{construct } Q(z) / \begin{cases} \text{Taylor } [Q(z)]_{z \rightarrow 0+} \hat{=} \sum_{n=0}^{\infty} a_n z^n \\ Q(z=1) = Q_{\text{phys}} \end{cases}$$

$$\{a_n\} \xrightarrow{\text{resummation}} Q(1)$$

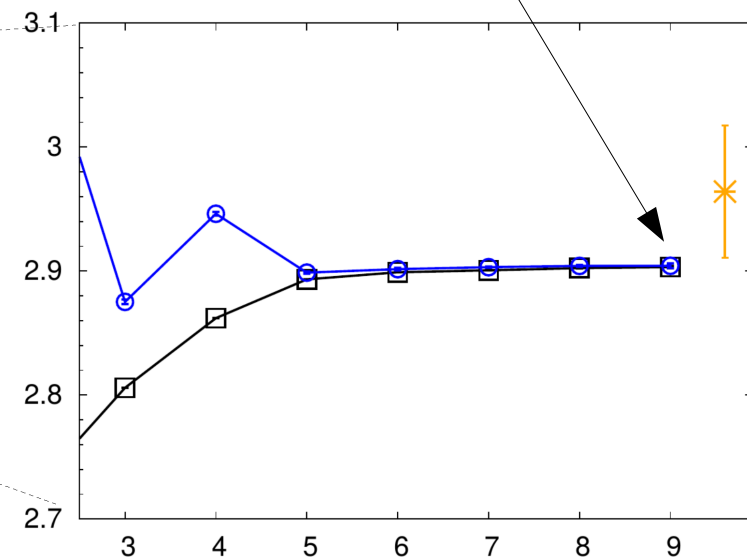
- Instanton method $\longrightarrow$ large-order behavior, branch cuts of $Q(z)$
- Nevanlinna theorem

Equation of state

$$\mu = 0 \quad \left(\frac{T}{T_F} \approx 0.6 \right)$$



precision < 0.1%



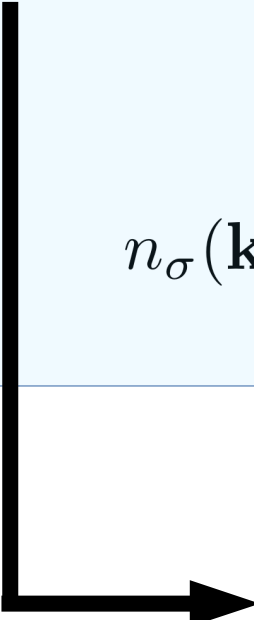
Contact parameter $\mathcal{C}$

$$\langle \hat{n}_{\uparrow}(\mathbf{r}) \hat{n}_{\downarrow}(\mathbf{0}) \rangle \underset{r \rightarrow 0}{\sim} \frac{\mathcal{C}}{(4\pi r)^2}$$

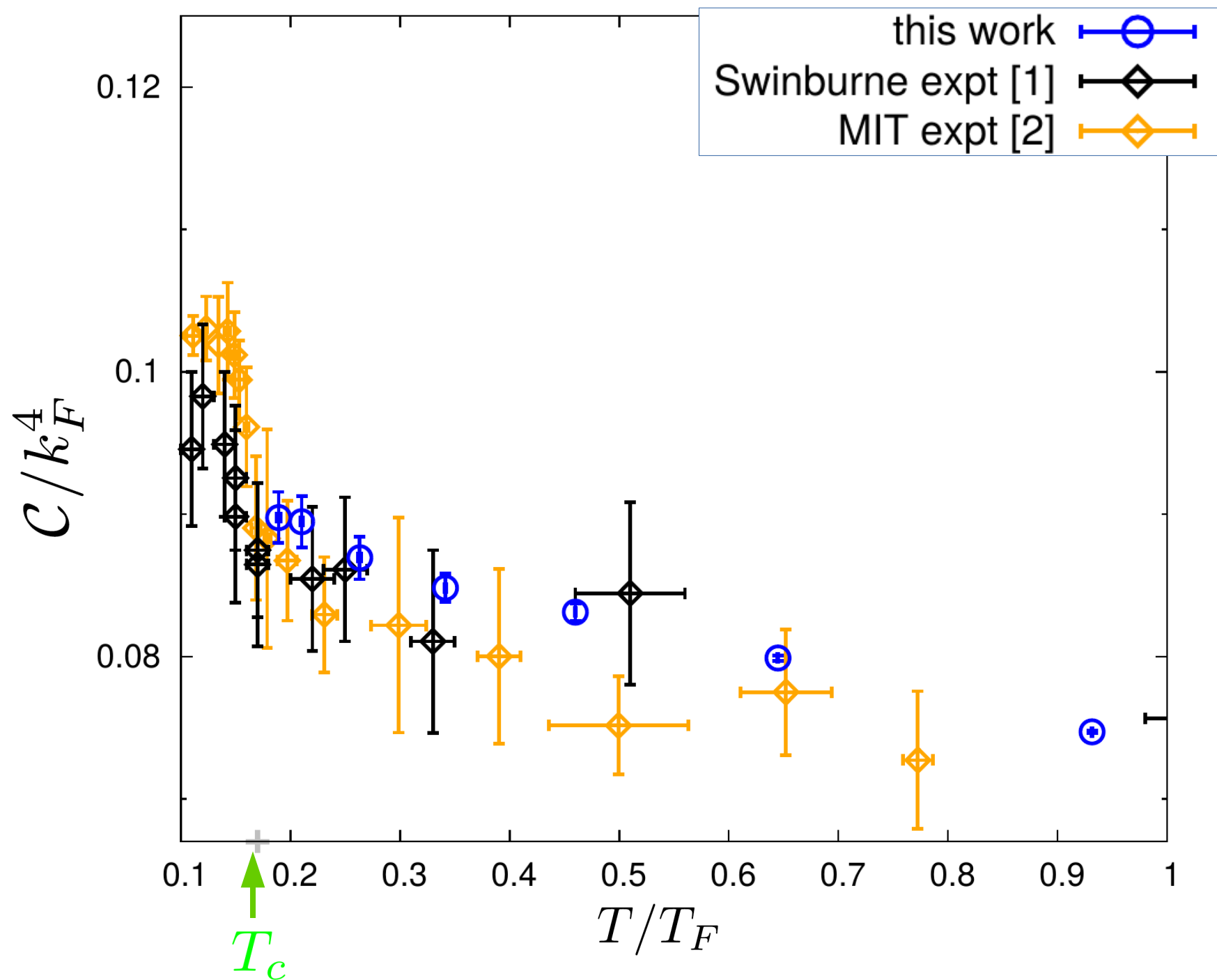
Measure all particle positions, in a unit volume.
Number of pairs of separation $< \epsilon$

$$\underset{\epsilon \rightarrow 0}{\sim} \mathcal{C} \epsilon \frac{1}{4\pi}$$

$$n_{\sigma}(\mathbf{k}) \underset{k \rightarrow \infty}{\sim} \frac{\mathcal{C}}{k^4} \quad \mathcal{C} = \frac{4\pi m}{\hbar^2} \left. \frac{\partial p}{\partial(1/a)} \right|_{T, \mu} \quad [\text{S. Tan, Ann. Phys. 2008}]$$

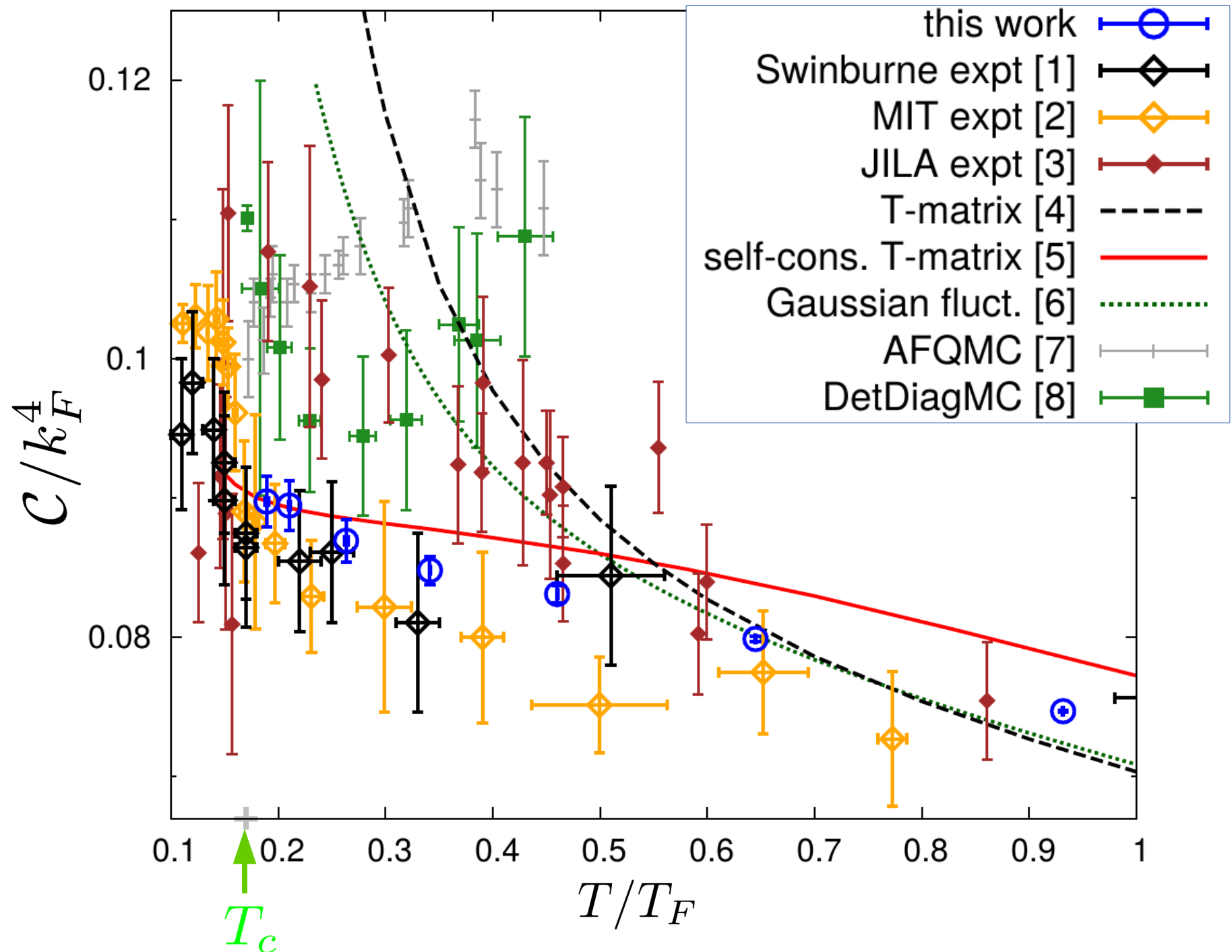


$$\mathcal{C} = -\Gamma(\mathbf{r} = \mathbf{0}, \tau = 0^-)$$



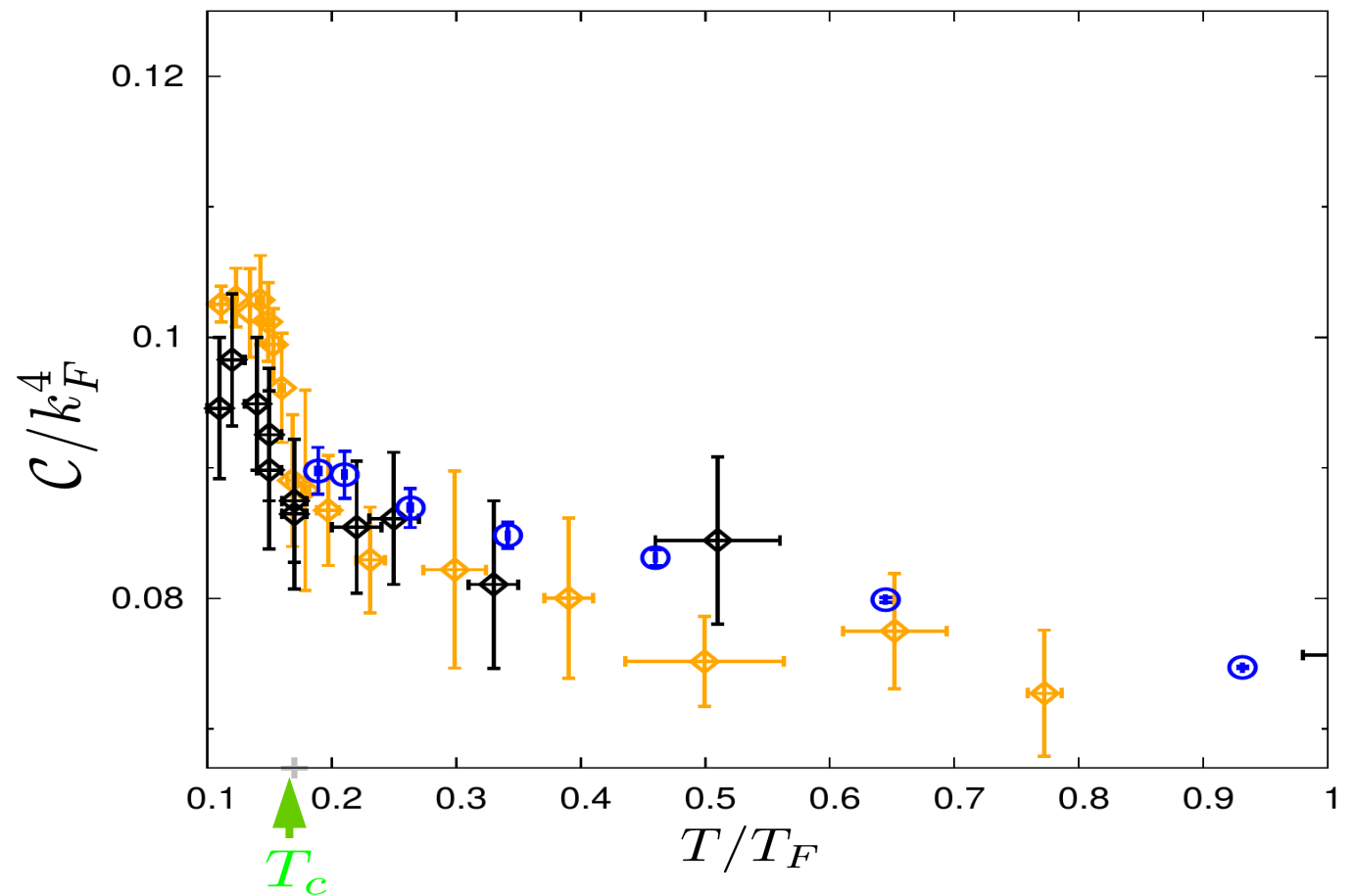
[1] Carcy, Hoinka, Lingham, Dyke, Kuhn, Hu, Vale, PRL 2019

[2] Mukherjee, Patel, Yan, Fletcher, Struck, Zwierlein, PRL 2019



[1] Vale group '19	[3] Jin group '12	[5] Zwerger group '11	[7] Drut et al. '11
[2] Zwierlein group '19	[4] Strinati group '10	[6] Hu et al. '11	[8] Goulko - Wingate '16

Critical behavior



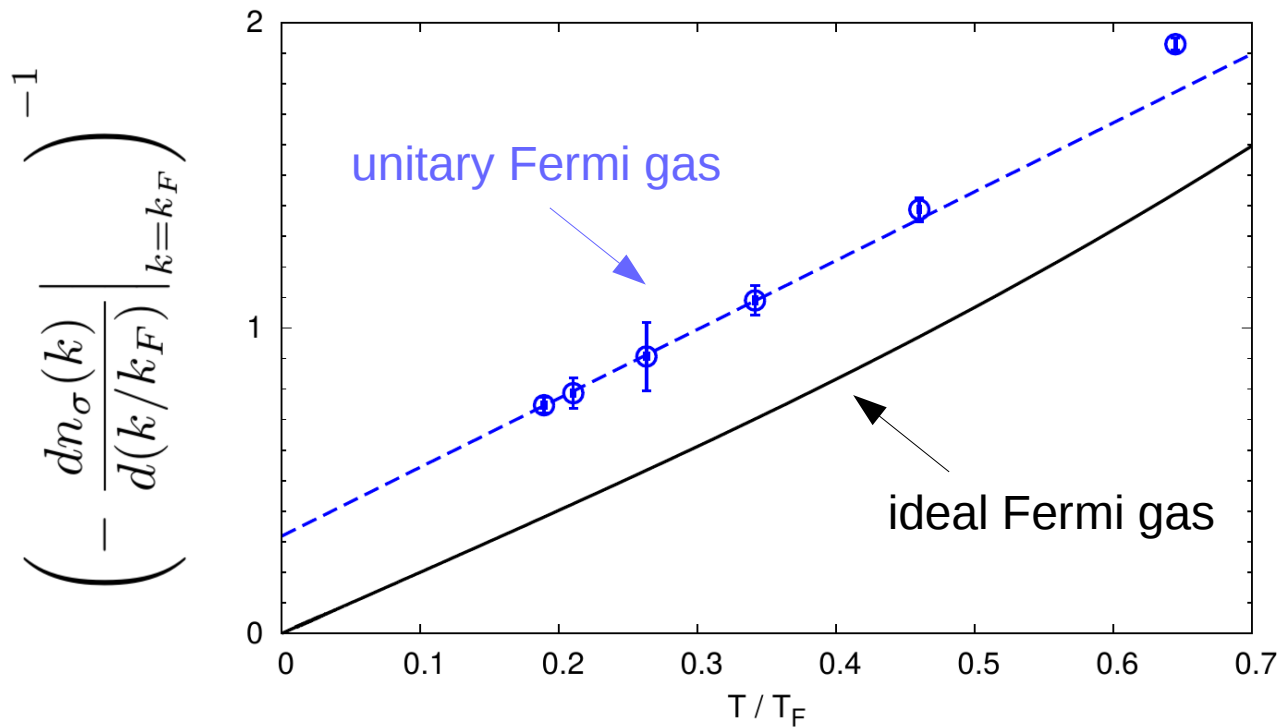
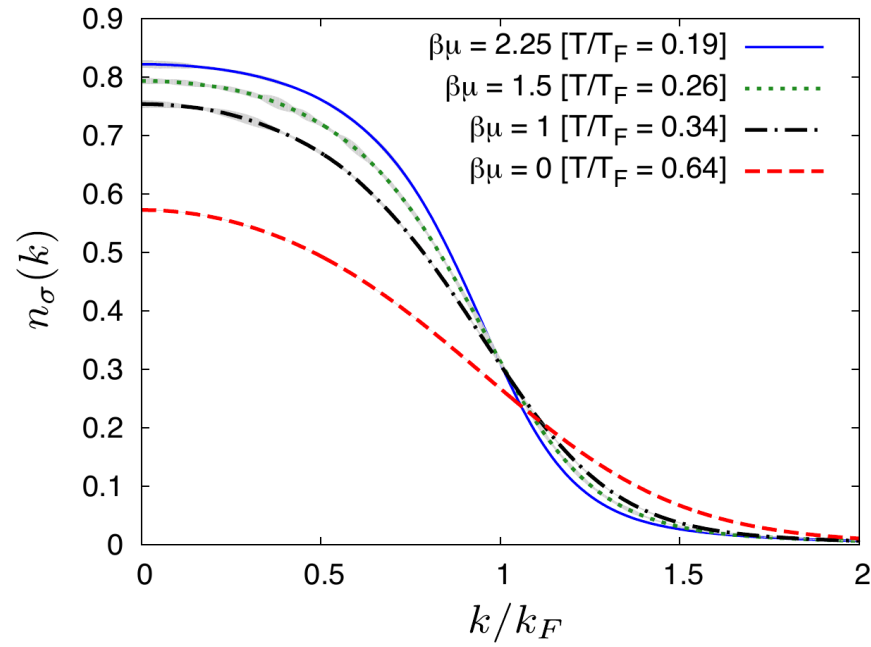
$$\mathcal{C}_{\text{singular}}(T) \sim \pm A_{\pm} |T - T_c|^{1-\alpha} \quad (T \rightarrow T_c^{\pm})$$

$$\alpha \simeq -0.015 \quad \frac{A_+}{A_-} \simeq 1.06$$

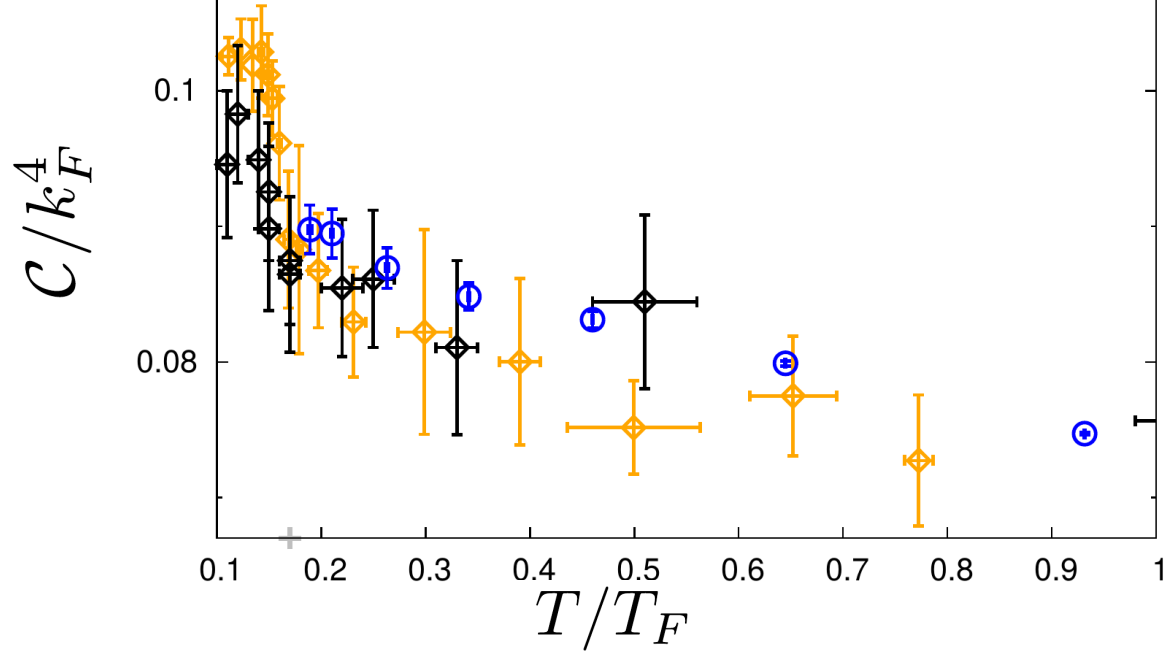
$\Rightarrow$ nearly $\propto (T - T_c)$

unlikely to see anything

Momentum distribution

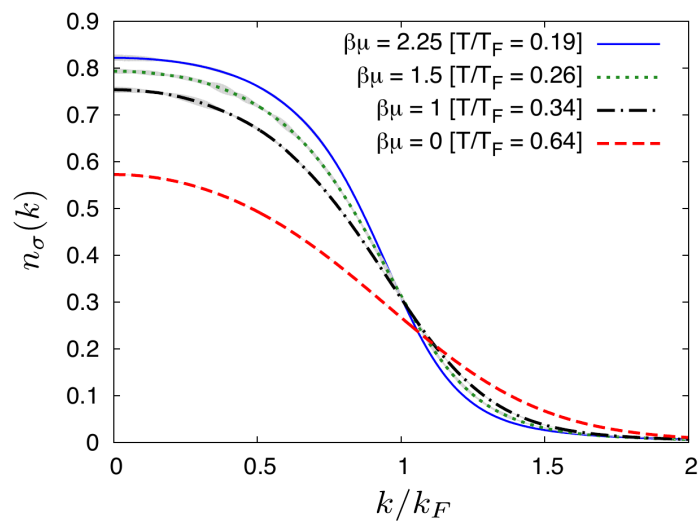


non
Fermi liquid
behavior

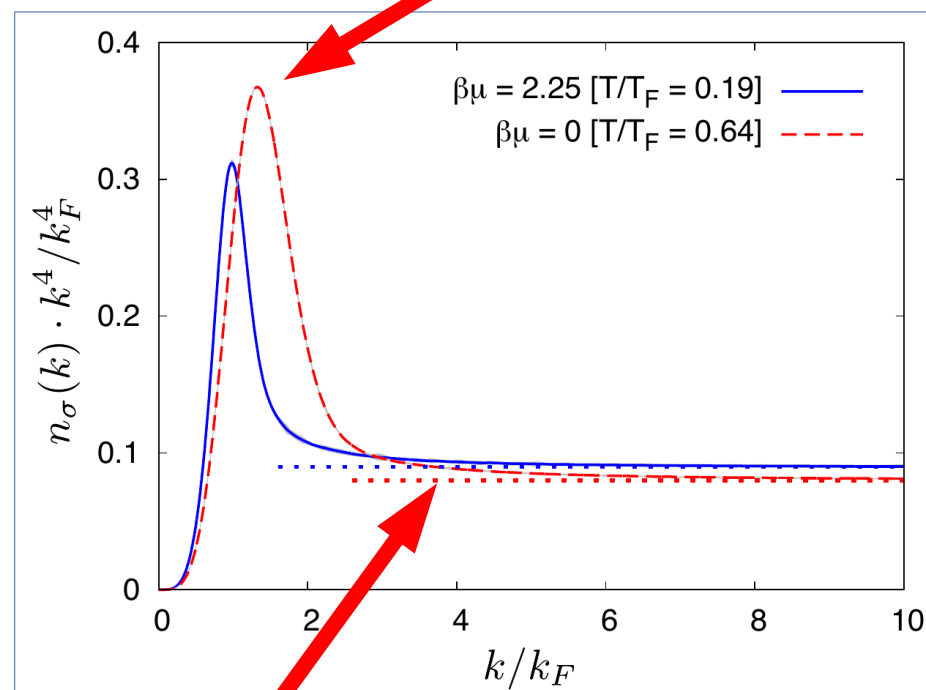


Why so flat?

Two competing effects:
at higher T ,



maximum is higher



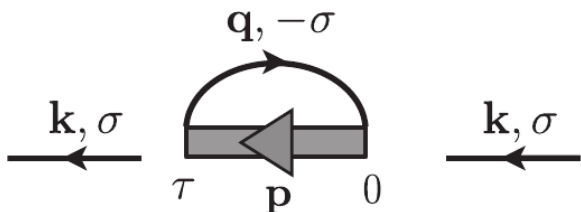
plateau sets in later $k \gg k_F$ and $1/\lambda$

Technical aspect: treatment of singular parts

Problem:

$$\Sigma_{\sigma}(k, \tau) \underset{\tau \rightarrow 0^+}{\sim} -4\sqrt{\frac{\pi}{\tau}} n_{-\sigma}$$

$$\begin{aligned} \Sigma_{\sigma}(k, \tau) \underset{k \rightarrow \infty}{\sim} & -4\sqrt{\frac{\pi}{\tau}} n_{-\sigma} e^{-(k^2/4)\tau}, & \tau \rightarrow 0^+ \\ & -e^{-(k^2/2)(\beta-\tau)}, & \tau \rightarrow \beta^- \end{aligned}$$

comes entirely from $\Sigma_{bold}^{(1)} =$ 

Solution:

$$\Sigma = \underbrace{\Sigma_{bold}^{(1)}}_{\text{analytically} + \text{accurate FT}} + \underbrace{[\text{other diagrams}]}_{\text{MC}}$$

works also
for **ladder**
scheme

$$G^{(N)} = \sum_{M=1}^N G^{(0)} \Sigma^{(M)} G^{(N-M)}$$

$$\Gamma^{(N)} = \sum_{M=1}^N \Gamma^{(0)} \Pi^{(M)} \Gamma^{(N-M)},$$

$$\Sigma_{1,bold;\sigma}^{(N)}(r, \tau) = \sum_{M=1}^N \Gamma^{(M-1)}(r, \tau) G_{-\sigma}^{(N-M)}(r, -\tau)$$

compute recursively over N

Polarized gas

algorithm

OUTLOOK

- Polarized gas:
compare with Fermi-liquid theory
FFLO correlations
 T_c
- other observables (density-density, spin susceptibility...)
- Analytic continuation $\rightarrow A(k, \omega)$ (radio-frequency)
 $S(k, \omega)$ (Bragg)
- Superfluid phases